



## A Cross-National Comparison of Turkish and American Mathematics Textbooks in Terms of Fraction Division Task Contexts

Research Article

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### ABSTRACT

The purpose of this study was to analyze and compare Turkish and the US mathematics textbooks with respect to contexts involved in fraction division tasks. To this end, three sixth grade mathematics textbooks from Turkey (TR1, TR2, and TR3 were published by the Ministry of National Education, Sevgi Press, and Dikey Press, respectively) and three sixth grade mathematics textbooks from the US (US1 and US2 were published by McGraw-Hill Education and US3 was published by Pearson Scott Foresman) were selected considering their up-to-dateness and representativeness. Textbooks were compared based on the number and variety of fraction division task contexts. The findings showed that the number and context of fraction division tasks varied to a considerable extent both within and across the selected two education systems. More than half of the Turkish textbook tasks and only a quarter of the US textbook tasks were contextual. The US textbooks included more varied task contexts compared to Turkish textbooks. More specifically, food and beverages context was existent in all textbooks. Shopping context was non-existent in the US textbooks, while student dependent contexts, sports, land, painting, animals, geography, free time activities, recycling, auto accessories, and room contexts were absent from Turkish textbooks. The implications for textbooks developers in providing students more quality learning opportunities in terms of fraction division contexts were discussed.

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**Keywords:**

Comparative studies, textbook analysis, fraction division, task context

### Introduction

Textbooks are generally major sources for teachers and students (Cady, Meier & Lubinski, 2006). In all countries, regardless of having a centralized or decentralized education system, curricular intentions of official school curricula are delineated by textbooks (Li, Chen & An, 2009). Besides, textbooks inform teachers about

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the content, scope, and sequence of topics taught and learned in the classrooms, illustrate exemplary conceptions/misconceptions of students, and provide teachers with specific classroom discussion techniques (Elsaleh, 2010). It can be argued that textbooks act as a bridge between the “intended curriculum” and the “implemented curriculum”. Hence, textbooks might be regarded as potentially implemented curricula (Son, 2012).

Howson (1995) addresses the significance of analyzing textbooks as follows: “textbooks exert a considerable influence on the teaching and learning of mathematics, so an understanding of how textbooks vary in their content and approach across countries is an important area of investigation” (pp. 5-6). Kulm and Capraro (2008) claim that there is a relationship between textbook quality and middle school students’ mathematics achievement in that the interplay with textbooks influences students’ knowledge of the content and their beliefs about teaching and learning of mathematics.

Similarly, there seems to be a significant relationship between student performance and the type of context based tasks included in textbooks (Wijaya, van den Heuvel-Panhuizen, Doorman & Robitzsch, 2014). However, there are different views on what a context means. For instance, Van den Heuvel-Panhuizen (2005) introduced the following two meanings of ‘context’ when it is used in an educational setting: ‘the learning environment’ and ‘a characteristic of a task presented to the students’. She described the learning environment context as an environment in which learning takes place and task context as “the words and pictures that help the students to understand the task, or concerning the situation or event in which the task is situated” (p. 2). In PISA, the term context was defined broadly as “a specific situation which includes all detailed elements used to formulate the problem (OECD, 2003, p. 32). In the current study, contextual tasks refer to tasks that are presented with illustrations such as pictures, representations, or real life situations and non-contextual tasks refer to tasks that are presented purely mathematically. It is important to note that although Van den Heuvel-Panhuizen (2005) argued that there is a big difference between word problems and context problems, in the current study, these two types of problems were considered synonymous to each other.

Contexts are known as important tools for teaching and learning mathematics since they provide different opportunities for students to learn (Wijaya, van den Heuvel-Panhuizen & Doorman, 2015a). Students’ experiences with context help them understand the mathematical topics conceptually and in richer way (Cooper & Harries, 2002). Contexts improve students’ accessibility to mathematical problems, increase transparency and elasticity of mathematical problems, and offer different strategies in solving mathematical problems (Van den Heuvel-Panhuizen, 2005). Besides, upon analysis of addition and subtraction word problems in American and Soviet elementary mathematics textbooks, Stigler, Fuson, Ham and Kim (1986) found out that contextual features of textbooks have impact on students’ proficiency in solving mathematical tasks. Furthermore, Gu, Huang and Marton (2004) pointed to the role of real life contexts and they stressed that increasing use of tasks that are set in a real life context in mathematics classrooms may give rise to an atmosphere which facilitates higher level understanding.

Although, the main goal of mathematics education is to help students apply mathematics in different contexts in real life and to help them solve context-based tasks easily (Boaler, 1993; Grauman, 2011), prior research has shown that many students display low performance in solving such tasks (e.g., Sam, Lourdusamy & Ghazali, 2001; Schwarzkopf, 2007). For instance, Wijaya et al. (2015a) examined the opportunities provided by 8th grade Indonesian textbooks to the students for solving context-based mathematical tasks related to equations of straight lines and they found out that only about 10% of the tasks included in the textbooks were contextual and that the students had great difficulty solving such tasks. Wijaya et al. (2015a) concluded that the lack of opportunity-to-learn provided by the textbooks for solving context-based tasks (i.e., the low percentage of context-based tasks in textbooks) might in part explain students’ difficulties in solving such tasks. In another study, Wijaya, van den Heuvel-Panhuizen and Doorman (2015b) explained that students

have four types of errors in solving context-based tasks as comprehension error (inability to understand the contextual task), transformation error (inability to identify correct mathematical procedures for solving a task), mathematical processing error (making mistakes when performing mathematical procedures), and encoding error (responses that are unrealistic and that do not match the real-world situation explained in the task). Therefore, it may be crucial for textbooks to include ample number and variety of context-based tasks in order to improve students' understanding of mathematics. To achieve this goal, context-based tasks should exemplify different context types (e.g., camouflage context, relevant and essential context, and no-context), different purposes (e.g., application and modeling), different types of information (e.g., matching, missing, superfluous), and different cognitive demand types (e.g., reproduction, connection, and reflection) (Wijaya et al., 2015a).

Context-based fraction tasks may also play an important role in students' understanding of fraction concepts and operations. The motivation for focusing on the topic of fractions, division of fractions in particular, among many mathematical topics is grounded in several perspectives. First, so far, textbook comparison studies have dealt with a quite limited range of mathematical content topics such as whole number addition and subtraction, whole number multiplication and division, decimals, integer addition and subtraction, functions, and complex numbers (Son & Senk, 2010). Although fraction concepts and operations play an important role in the teaching of mathematics, textbook comparison studies that focus on fractions are very few (e.g., Li et al., 2009; Son, 2012). Besides, fractions are widely used in mathematics education and have great importance in other disciplines as well (Ben-Chaim, Keret & Ilany, 2012). For instance, fractions form the basis of introductory mathematics and other mathematical topics such as algebra and probability (Clarke & Roche, 2009). However, fractions are notorious for the difficulty encountered not only by students (e.g., Ni, 2001; Vamvakoussi & Vosniadou, 2010) but also by teachers (e.g., An, Kulm & Wu, 2004; Izsak, 2008; Ma, 1999; Tirosh, 2000). As Lamon (2007) expressed, fractions "arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites" (p. 629). This difficulty on the part of teachers may stem in part from conceptual richness of fractions (Li & Kulm, 2008). That is, teaching fractions conceptually requires making connections with other mathematical knowledge and employing different representations and real life contexts (Li, 2008). Moreover, teaching fractions in an inappropriate way or procedurally may lead to student misconceptions and consequently may inhibit their understanding of future concepts related to fractions (Kazemi & Rafiepour, 2018).

In particular, fraction division is the most complex operation for elementary school students (Ma, 1999; Post, Harel, Behr & Lesh, 1991). Teaching fraction operations through "procedure-oriented, memory-based instruction" and "attributing little meaning to such operations" can be considered the main reason for this complexity (Son & Senk, 2010). After examining a number of school mathematics curricula, pre-service teacher education textbooks and materials, and the research on division of fractions, Son and Crespo (2009) revealed that 'invert and multiply' ( $a/b \div c/d = a/b \times d/c$ ) and 'using a unit rate' ( $a/b \div c/d = (a/b \times d/c) \div (c/d \times d/c)$ ) were the most common formal strategies (i.e., strategies that do not develop naturally) taught to the US students. Teaching the invert and multiply strategy procedurally is very easy for teachers (Li et al., 2009). However, teaching in this way makes it the most mechanical and the least understood strategy by elementary or middle school students (National Council of Teachers of Mathematics [NCTM], 2000). On the other hand, teaching the invert and multiply strategy conceptually is very challenging for teachers and for students to understand (Huang, Yetkiner Özel, Li & Osborne, 2014). Conceptual teaching of invert and multiply strategy is also a very challenging task for Turkish mathematics teachers since the middle school mathematics curriculum (Ministry of National Education [MoNE], 2013) does not explicitly specify multiplicative inverse property as a core concept for fraction division algorithm (i.e., fraction division refers to multiplying the

dividend by the multiplicative inverse of the divisor). Thus, textbooks' provision of mathematical justifications and concrete or visual demonstrations is of crucial importance in helping students learn fraction division conceptually (Li, 2008).

To sum up, the association between textbook quality and students' mathematics performance, the association between student performance and the type of context based tasks included in textbooks, the importance of fractions in understanding various mathematical topics, and the challenges faced by teachers in teaching division of fractions conceptually to the students make it crucial to examine textbooks in terms of context-based tasks related to division of fractions. Despite the significance of real life contexts in school mathematics and the widespread consensus in mathematics education community on linking school mathematics to the real world (e.g., Gainsburg, 2008), very few studies have been conducted on this area (e.g., Chapman, 2006; Verschaffel, Greer & De Corte, 2000). Besides, fraction division has received little attention in cross-national textbook comparison studies despite being crucial (Son & Senk, 2010). Thereby, in this study I attempted to analyze the extent of variation in sixth grade mathematics textbooks in terms of fraction division contexts both within and across Turkey and the US. More specifically, the following research questions guided this study:

1. What type of contexts do sixth grade mathematics textbooks present in covering division of fractions both within and across the two education systems?
2. What is the extent of variation within and across the US and Turkish mathematics textbooks in terms of fraction division contexts?

It is expected that answering the aforementioned questions may, to some extent, shed light on the learning opportunities provided to the students by the textbooks from Turkey and the US in learning division of fractions. By this way, the potential strengths and weaknesses of textbooks in introducing and developing the notion of fraction division may be uncovered. Consequently, as argued by Cady, Collins and Hodges (2015), it may be helpful in making deliberate decisions about mathematics instruction.

### **Methodology**

The focus of this study was to examine the variety of contexts included in the fraction division tasks covered by the US and Turkish sixth grade mathematics textbooks. In the following parts, the selected mathematics textbooks and data analysis procedures are explained.

#### **Textbooks Analyzed**

In this study, I compared only the student editions of sixth grade mathematics textbooks-three from Turkey and three from the US. Neither teacher guidebooks nor student workbooks were included in the analysis. The rationale for focusing on student textbooks is that the current study investigates the learning opportunities provided to the students directly by the textbooks rather than the opportunities provided by teachers during their classroom practice. Besides, student textbooks are main resources that include explanations, examples, and activities related to different mathematical topics and students directly resort to them, rather than workbooks, when learning new mathematical topics. Information about the mathematics textbooks selected for analysis in this study is presented in Table 1 (see Appendix for the bibliographic information about these textbooks).

**Table 1.** Turkish and American mathematics textbooks examined in this study

|        | Textbook title                    | Textbook code | Year published | Publisher                      |
|--------|-----------------------------------|---------------|----------------|--------------------------------|
| Turkey | Middle school mathematics grade 6 | TR1           | 2016           | Ministry of National Education |
|        | Middle school mathematics grade 6 | TR2           | 2016           | Sevgi Press                    |
|        | Middle school mathematics grade 6 | TR3           | 2015           | Dikey Press                    |
| US     | Everyday Mathematics 4            | US1           | 2015           | McGraw-Hill Education          |
|        | Glencoe Math Course 1             | US2           | 2014           | McGraw-Hill Education          |
|        | enVisionmath 2.0                  | US3           | 2016           | Pearson Scott Foresman         |

Turkey has a centralized education system and consequently has a national school mathematics curriculum released by the MoNE (2013). Thus, all of the Turkish mathematics textbooks were prepared in line with this curriculum, and middle grades mathematics teachers and students (grades 5-8) follow it during the teaching and learning of school mathematics. While TR1 was prepared by the MoNE, TR2 and TR3 were prepared by private publishers. In Turkey, textbooks are distributed at no cost to students enrolled in public schools regardless of their parents' income levels. For this reason, it is possible to say that most of the Turkish students have the chance to have access to TR1, TR2, and TR3. More specifically, TR1 and TR2 were used by sixth graders during the 2016-2017 school year, and TR3 was used by the them during the 2015-2016 school year. Therefore, the three Turkish textbooks examined in the current study were published recently and they are all up to date. According to the report of Dossey, McCrone and Halvorsen (2016), Pearson and McGraw-Hill are among the major publishers of textual materials used in the US. Moreover, the recent survey on science and mathematics education has shown that the textbooks published by these companies are among the most commonly used middle school mathematics materials (Fulkerson, 2013). The selected US textbooks belong to these major publishers. Thus, it is considered that these textbooks would better represent the fraction division task contexts available to the US students. Besides, they were published between years 2014 and 2016 (see Table 1). Thus, they are also up to date. Briefly speaking, Turkish and the US mathematics textbooks were selected considering their up-to-dateness and representativeness.

The middle school mathematics curriculum released by MoNE (2013) specifies 9 learning objectives for the teaching of fraction operations and the suggested time period for teaching them is 24 lesson hours. MoNE (2013) specifies 3 learning objectives for fraction division but does not specify any time period for its teaching. However, the ratio of total lesson hours to total learning objectives for the teaching of fraction operations shows that mathematics teachers need to allocate roughly 8 lesson hours to the teaching of fraction division. MoNE (2013) specifies the following 3 learning objectives for fraction division: divide a whole number by a unit fraction, divide a unit fraction by a whole number, and make sense of these operations; divide a whole number by a fraction, divide a fraction by a whole number, and make sense of these operations; divide a fraction by another fraction, and make sense of related operations.

The US has a decentralized education system and each state has its own standards. However, The Council of Chief State School Officers released the Common Core State Standards in 2010. By the year 2016, 43 of the 50 states and the District of Columbia adopted the common core state standards (Dossey et al., 2016). The selected American textbooks followed the goals specified in the Common Core State Standards for Mathematics (CCSSM) document (CCSSI, 2010). According to this document, sixth grade students are expected to use and expand prior understandings of whole number division in learning fraction divisions. More specifically, the following content goal is specified: "interpret and compute quotients of fractions, and

solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem" (CCSSI, 2010, p. 42). However, the CCSM document does not specify any time period for the teaching of fraction division in particular and the whole mathematical topics in general. Thus, this paves the way for variations between the US textbooks in terms of allocation, presentation, and organization of content depending upon the foci and priorities of textbook writers and publishers.

This study analyzed the contexts of the fraction division tasks covered by the six mathematics textbooks. By the term task, I refer to worked out and non-worked out fraction division examples. Worked out examples refer to examples that are used in explaining the topic of fraction division. Shortly, they are explanatory examples. Non-worked out examples are exercise examples which are left to the students to solve in order to practice and consolidate their fraction division ideas. Some of the selected textbooks included activities and some of them did not. Besides, the meanings given to activities by the two countries were different from each other. Thus, textbook activities were not included into the analysis process.

### Data Analysis

Initially, I started analyzing textbooks by considering the analytic coding scheme of Cady et al. (2015). However, it was necessary to revise and refine the problem context category of this coding scheme due to the fact that the textbooks examined in the current study are different from those of Cady et al. (2015) and not surprisingly the textbooks selected by Cady et al. (2015) present different contexts for covering division of fractions. Namely, Cady et al. (2015) examined textbooks entitled *Connected Mathematics Project*, *Math Thematics*, and *Glencoe Mathematics: Applications and Connections* and they used the following codes for the fraction contexts included in these three middle grades textbooks: "cooking, science, shopping, money, sharing, sports, discrete mixtures-fruit/nut mix, continuous mixtures-liquids, no context (naked), and other" (p. 109). For instance, I used the code "foods and beverages" instead of "cooking". In addition, I preferred not to use the code "sharing" since this code had to do more with the meaning given to the fraction division tasks rather than the context embedded in the tasks. Apart from these, I derived some new codes such as "carpentry" and "gardening". Ultimately, in some fraction division tasks, the students were expected to pose real life problems by using the given division operation. These types of tasks were coded as "student dependent contexts".

All mathematics textbooks selected for this study were initially analyzed in their original languages and Turkish mathematics textbook tasks were translated into English when necessary. First, case-by-case analysis was conducted. Namely, the textbooks were first coded separately by the author and another mathematics educator who are fluent both in English and Turkish. The inter-coder reliability, "the extent to which the different judges tend to assign exactly the same rating to each object" (Tinsley & Weiss, 2000, p. 98), for the whole coding was calculated using the percentage agreement index and the agreement between the two coders was found to be as 92%. After the coding process, the two coders discussed the coding categories comprehensively and the conflicts between the two coders were resolved in a number of meetings. Next, cross-case analysis of fraction division tasks was performed by comparing the emergent codes across the six textbooks. By this way, the same codes were assigned to the same contexts that are existent in more than one textbook. Besides, some codes were joint together under one category. For instance, the codes "foods" and "beverages" were renamed as "foods and beverages". Hence, case-by-case and cross-case analysis of fraction division tasks presented in the textbooks were completed in full consensus of the two coders.

### Results

In this study, three Turkish and three American sixth grade mathematics textbooks were examined and altogether 385 fraction division tasks were analyzed (see Table 2). Turkish textbooks included 66 fraction division tasks. In particular, TR1, TR2, and TR3 included 33, 18, and 15 fraction division tasks, respectively. Meanwhile, American textbooks included 319 fraction division tasks. More specifically, US1, US2, and US3

included 20, 169, and 130 fraction division tasks, respectively. Note hereafter that the term “task” will be used to refer to “fraction division task” for ease of clarity.

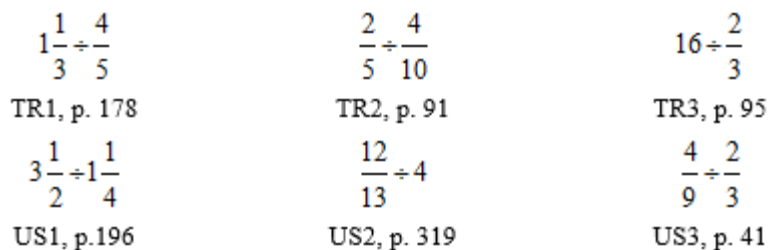
As given above, the number of tasks included in textbooks varied to a considerable extent both within and across the selected two education systems. Namely, the number of tasks included in TR1 was roughly twofold of the ones included in TR2 or TR3. The US textbooks presented more dramatic variations in terms of number of tasks. More explicitly, the number of tasks included in US2 was more than eightfold of the ones included in US1. Similarly, US3 included roughly six times as many tasks when compared to the number of tasks presented in US1. Overall, the most proportion of tasks came from US2 and US3. Namely, the total number of tasks included in these textbooks ( $n = 299$ ) constituted 77.66% of the whole tasks examined in this study.

However, it is important to note that slightly more than one third of the Turkish textbook tasks (37.87%) were non-contextual, while nearly three fourths of American textbook tasks (72.72%) were non-contextual. Overall, two third of the whole tasks (66.75%) were presented without any context. The breakdown of tasks with respect to context is presented in Table 2.

**Table 2.** The number of contextual and non-contextual tasks in Turkish and US textbooks

| Fraction division tasks | Sixth grade mathematics textbooks |     |     |          |     |     | Total |
|-------------------------|-----------------------------------|-----|-----|----------|-----|-----|-------|
|                         | Turkish                           |     |     | American |     |     |       |
|                         | TR1                               | TR2 | TR3 | US1      | US2 | US3 |       |
| Contextual tasks        | 27                                | 10  | 4   | 7        | 45  | 35  | 128   |
| Non-contextual tasks    | 6                                 | 8   | 11  | 13       | 124 | 95  | 257   |
| Total                   | 33                                | 18  | 15  | 20       | 169 | 130 | 385   |

As presented in Table 2, one third of the whole tasks examined in this study were presented with context (33.25%). Within Turkish textbooks, TR1 (81.81%) and TR2 (55.55%) gave more weight to contextual tasks, while TR3 (26.66%) did not. Overall, more than half of the tasks included in Turkish textbooks (41 out of 66 tasks or 62.12%) were contextual. However, the US textbooks paid less attention to the presentation of contextual tasks (87 out of 319 tasks or 27.28%). In more detail, 35.00% of the US1 tasks, 26.63% of the US2 tasks, and 26.92 % of the US3 tasks were contextual. Exemplary non-contextual tasks are presented in Figure 1.



**Figure 1.** Tasks presented without any context

The following section provides more detailed findings related to the contextual tasks presented in the US and Turkish textbooks. Besides, some illustrative examples for each context type are presented.

**Fraction Division Contexts Presented in the US and Turkish Textbooks**

The analysis of data showed that there were some variations in fraction division contexts to a considerable extent both within and across the selected two country textbooks (see Table 3). Turkish textbooks included eight different contexts altogether as foods and beverages, travelling, strips, handicrafts, shopping, gardening, carpentry, travelling, and household goods. In particular, gardening and household goods contexts

did not appear in TR1. While foods and beverages, handcrafts, strips, and gardening contexts were included in TR2, travelling, carpentry, shopping, and household goods contexts did not exist in it. Meanwhile, only foods and beverages and household goods contexts were included in TR3.

There was much more diversity in American textbooks in terms of contexts when compared to Turkish textbooks. American textbooks included nine different contexts in addition to the ones included in Turkish textbooks excluding shopping context. These contexts were sports, painting, animals, land, recycling, free time activities, geography, room, and auto accessory. Within American textbooks, US1 included only two different contexts as foods and beverages and handcrafts. US2 included twelve different contexts as foods and beverages, carpentry, strips, gardening, household goods, sports, land, painting, geography, animals, free time activities, and recycling. Finally, US3 included nine different contexts as foods and beverages, traveling, sports, carpentry, handcrafts, auto accessory, room, land, and painting. The fraction division context types included in the selected mathematics textbooks and their frequencies are presented in Table 3.

As presented in Table 3, foods and beverages was the only context that existed in all textbooks. Actually, 50 out of 128 contextual tasks (39.06%) involved foods and beverages. Within Turkish textbooks, TR1 (62.96%) and TR3 (75.00%) had a higher rate of foods and beverages tasks, while TR2 (20.00%) had a lower rate of such tasks. Overall, more than half of the contextual tasks included in Turkish textbooks (53.65%) involved foods and beverages. Within the US textbooks, US1 had a higher rate of foods and beverages tasks, while US2 and US3 had a similar and lower rate of such tasks (71.42%, 26.66%, and 31.42% of the contextual tasks for US1, US2, and US3, respectively).

**Table 3.** Fraction division contexts presented in Turkish and the US textbooks

| Fraction division contexts |                           | Sixth grade mathematics textbooks |     |     |          |     |     | Total |
|----------------------------|---------------------------|-----------------------------------|-----|-----|----------|-----|-----|-------|
|                            |                           | Turkish                           |     |     | American |     |     |       |
|                            |                           | TR1                               | TR2 | TR3 | US1      | US2 | US3 |       |
| TR and US                  | Foods and beverages       | 17                                | 2   | 3   | 5        | 12  | 11  | 50    |
|                            | Handcrafts                | 1                                 | 3   | -   | 2        | -   | 2   | 8     |
|                            | Travelling                | 2                                 | -   | -   | -        | -   | 5   | 7     |
|                            | Carpentry                 | 2                                 | -   | -   | -        | 2   | 2   | 6     |
|                            | Strips                    | 3                                 | 1   | -   | -        | 2   | -   | 6     |
|                            | Gardening                 | -                                 | 2   | -   | -        | 1   | -   | 3     |
|                            | Household goods           | -                                 | -   | 1   | -        | 1   | -   | 2     |
|                            | Others                    | -                                 | 2   | -   | -        | 4   | 4   | 10    |
| TR                         | Shopping                  | 2                                 | -   | -   | -        | -   | -   | 2     |
| US                         | Student dependent context | -                                 | -   | -   | -        | 8   | 1   | 9     |
|                            | Sports                    | -                                 | -   | -   | -        | 2   | 4   | 6     |
|                            | Land                      | -                                 | -   | -   | -        | 2   | 1   | 3     |
|                            | Painting                  | -                                 | -   | -   | -        | 2   | 1   | 3     |
|                            | Animals                   | -                                 | -   | -   | -        | 3   | -   | 3     |
|                            | Geography                 | -                                 | -   | -   | -        | 2   | -   | 2     |
|                            | Free time activities      | -                                 | -   | -   | -        | 2   | -   | 2     |
|                            | Recycling                 | -                                 | -   | -   | -        | 2   | -   | 2     |
|                            | Auto accessories          | -                                 | -   | -   | -        | -   | 2   | 2     |
|                            | Rooms                     | -                                 | -   | -   | -        | -   | 2   | 2     |
|                            | Total                     | 27                                | 10  | 4   | 7        | 45  | 35  | 128   |

Altogether, nearly one third of the contextual tasks included in the US textbooks (32.18%) involved foods and beverages. Thus, contextual tasks involving foods and beverages were more prevalent in Turkish



textbooks compared to the US textbooks. Some illustrative tasks involving foods and beverages context are presented in Figure 2.

Yaş pastanın bir dilimi, bütün yaş pastanın  $\frac{1}{8}$ 'idir. Bu yaş pastanın  $\frac{3}{4}$ 'ü yenirse kaç dilim yaş pasta kalacağını bulalım:

İşlemi yapalım:  $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot \frac{8}{1} = \frac{24}{4} = 6$  dilim yaş pasta yenilmiştir.

Bir bütün pasta 8 dilime ayrılmıştı,  $8 - 6 = 2$  dilim yaş pasta kalmıştır.

[A slice of a cake is  $\frac{1}{8}$  of the whole cake. Let's find how many slices will remain if  $\frac{3}{4}$  of the whole cake is eaten. Let's do the operation:  $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \cdot \frac{8}{1} = \frac{24}{4} = 6$  slices of a cake was eaten. A whole cake was cut into 8 slices.  $8 - 6 = 2$  slices will remain]

TR3, p. 93

Regina has 9 pizzas. If each person can eat  $\frac{1}{2}$  of a pizza, how many people can Regina serve?

US1, p. 195



Figure 2. Tasks involving foods and beverages in TR3 and US1

Less than one tenth of the contextual tasks examined in this study involved handcrafts (6.25%). Within Turkish textbooks, TR1 ( $n = 1$ ) and TR2 ( $n = 3$ ) included a few handcrafts tasks, while such tasks were not existent in TR3. Altogether, almost one tenth of the contextual tasks (9.75%) presented in Turkish textbooks were related to handcrafts. Within the US textbooks, US1 and US3 included few but the same number of handcrafts tasks ( $n = 2$  for each textbook), while US2 did not include any such task. All told, less than 5% of the contextual tasks included in the US textbooks (4.59%) involved handcrafts. Hence, tasks comprising handcrafts context were more frequent in Turkish textbooks compared to the US textbooks. An exemplary task related to handcrafts context is presented in Figure 3.

2. Melike teyze, bir yumak ipin  $\frac{8}{9}$ 'lük kısmının  $\frac{1}{9}$ 'u ile bir çift çorap örüyor. Melike teyze, yumağın  $\frac{8}{9}$ 'lük kısmından kaç çift çorap örebilecektir?

[Aunt Melike knits a pair of socks with  $\frac{1}{9}$  of a yarn. How many pairs of socks can Aunt Melike knit with  $\frac{8}{9}$  of the yarn?]



Figure 3. Handcrafts task in TR2, p. 91

Roughly, one twentieth of the contextual tasks involved travelling (5.46%). Tasks involving this context were presented only by TR1 ( $n = 2$ ) and US3 ( $n = 5$ ). However, the percentage of travelling tasks in US3 (14.28%) almost doubled that of TR1 (7.40%). An exemplary task related to travelling context is presented in Figure 4.

Kayla drives her new car to work every day. It uses  $1\frac{2}{3}$  gallons of gas for each round trip. How many round trips to work can Kayla drive on a full tank of gas?



Figure 4. Traveling task in US3, p. 45

Less than 5% of the entire contextual tasks involved carpentry (4.68%). Within Turkish textbooks, only TR1 ( $n = 2$ ) presented tasks with carpentry context and within the US textbooks, US2 ( $n = 2$ ) and US3 ( $n = 2$ ) presented tasks with such a context. However, the percentage of carpentry tasks in TR1 (7.40%) almost doubled that of US2 (4.44%) and was considerably higher than that of US3 (5.71%). Similarly, tasks involving strips context were less than one twentieth of the contextual tasks included in all textbooks (4.68%). Within Turkish textbooks, TR1 ( $n = 3$ ) and TR2 ( $n = 1$ ) included strips tasks and within the US textbooks, only US2 ( $n = 2$ ) included such tasks. Meanwhile, the percentage of strips tasks in TR1 (11.11%) and TR2 (10.00%) was more than twofold of the percentage of strips tasks in US2 (4.44%). To illustrate, tasks involving strips and carpentry contexts are presented in Figure 5.

Jalisa is using  $\frac{5}{6}$  yard of ribbon to make bows for her party favors. Jalisa needs to make 6 bows. What is the length of the ribbon used for each bow?  
US2, p. 323

Mr. Roberts has a board that is 3 feet long. He plans to cut the board into pieces that are each  $\frac{3}{4}$  foot long to build a set of shelves. How many shelves can he make?  
US3, p. 32

**Use Structure** How many  $\frac{3}{4}$ s are in 3?

Figure 5. Strips task in US2 and carpentry task in US3

Gardening and household goods were among the contexts that were considerably less apparent in Turkish and the US textbooks. In more detail, only one textbook from each country presented tasks involving gardening ( $n = 2$  for TR2 and  $n = 1$  for US2) and household goods ( $n = 1$  for TR3 and  $n = 1$  for US2). Example tasks involving gardening and household goods contexts are presented in Figure 6.

**MP Reason Abstractly** Carlota has  $\frac{3}{4}$  ton of mulch she is going to divide evenly among 5 flower beds. How much mulch will each flower bed contain?  
US2, p. 321

Sema, uzunluğu  $5\frac{2}{5}$  m olan halıyı adımları ile ölçüyor. Sema'nın bir adımının uzunluğu  $\frac{3}{5}$  m olduğuna göre bu halının uzunluğu, Sema'nın adımları ile kaç adımdır?  
[Sema measures the length of a carpet in footsteps. The carpet and her foot is  $5\frac{2}{5}$  meters and  $\frac{3}{5}$  meters long, respectively. How many footsteps long is the carpet?]  
TR3, p. 96

Figure 6. Tasks involving gardening in US2 and household goods in TR3

When a context appeared in one textbook and if no more than one task included that context, such contextual tasks were classified under "others" category. That is, there was only one task for each of the

following contexts within the examined textbooks: tailoring, jewellery, aquatic store, metals, hauling, herbs, tiling, selling, and genetics.

Thus far, findings related to the contexts that were apparent both in Turkish and the US textbooks (i.e., foods and beverages, handcrafts, traveling, carpentry, strips, gardening, and household goods contexts) are presented. Findings related to the contexts that were included only in Turkish textbooks (i.e., shopping) and only in American textbooks (i.e., sports, land, painting, animals, auto accessories, rooms, geography, free time activities, and recycling contexts) are explained as follows.

It is worth noting that only one of the Turkish textbooks, namely TR1, presented shopping tasks ( $n = 2$ ), while none of the US textbooks included such tasks. An example task involving shopping context is presented in Figure 7.

Bir çiçekçide bulunan çiçeklerin  $\frac{3}{5}$ 'ü güldür. Çiçekçi her gün eşit sayıda sattığı gülleri 4 günde bitiriyor. Bu durumda 1 günde satılan gül miktarı toplam çiçeklerin ne kadarıdır?

[In a flower store,  $\frac{3}{5}$  of whole flowers are roses. If the florist sells the same number of roses each day, he sells all of them in 4 days. Then, what fraction of the whole flowers are sold in 1 day?]

Figure 7. Shopping task included in TR1, p. 173

In two of the US textbooks, (i.e., US2 and US3), some tasks asked students to pose real life problems by using the given division operations. These types of tasks referred to student dependent contexts since the students were free to select their own contexts when posing fraction division problems. Overall, less than one tenth of the contextual tasks (7.03%) included student dependent contexts. In more detail, student dependent contexts were far more dominant in US2 ( $n = 8$ , %17.77) than in US3 ( $n = 1$ , 2.85%). Examples for student dependent contexts included in US2 and US3 are presented in Figure 8.

**MP Model with Mathematics** Write a story context that involves  $4 \div \frac{4}{5}$ .  
Solve the problem and multiply to check your answer.

US2, p. 304

Write a problem that could be solved by finding  $\frac{5}{8} \div \frac{2}{5}$ .

US3, p. 42

Figure 8. Tasks involving student dependent contexts in US2 and US3

Tasks involving sports, land, and painting contexts were included in US2 and US3, while such tasks were not existent in US1. The rate of sports tasks in US3 ( $n = 4$ , 11.42%) was higher than that of US2 ( $n = 2$ , 4.44%) indicating that sports tasks were more apparent in US3 compared to US2. On the other hand, land and painting contexts were less visible in US3 compared to US2. Namely, US3 included only one task for each of the land and painting contexts, while US2 included two tasks for each of them. Tasks that exemplify sports, land, and painting contexts are presented in Figure 9.

**At summer camp, the duration of a field hockey game is  $\frac{3}{4}$  hour. The camp counselors have set aside 6 hours for field hockey games. How many games can be played?**

Divide 6 by three-fourths.

$$6 \div \frac{3}{4} = \frac{6}{1} \times \frac{4}{3}$$

Multiply by the reciprocal.

$$= \frac{\overset{2}{\cancel{6}}}{1} \times \frac{4}{\underset{1}{\cancel{3}}}$$

Divide 3 and 6 by the GCF, 3.

$$= \frac{8}{1} \text{ or } 8$$

Simplify.

So, 8 games can be played.

US2, p. 308

How wide is a rectangular strip of land with a length of  $\frac{3}{4}$  mile and an area of  $\frac{1}{2}$  square mile? Use the area formula:  $A = \ell \times w$ .

$$\frac{1}{2} = \frac{3}{4}w \rightarrow \frac{1}{2} \div \frac{3}{4} = w \rightarrow \frac{1}{2} \times \frac{4}{3} = w \rightarrow w = \frac{2}{3}$$

The strip of land is  $\frac{2}{3}$  mile wide.

$\frac{3}{4}$  mi  $A = \frac{1}{2}$  mi<sup>2</sup>

US3, p. 39

The area of a rectangular painting is  $\frac{1}{6}$  square yard. The width is  $\frac{2}{3}$  yard. What is the length of the painting? Use the formula  $A = \ell \times w$ .

US3, p. 41

Figure 9. Tasks involving sports in US2, land and painting in US3

Tasks involving animals, geography, free time activities, and recycling context were existent only in US2. Besides, in this textbook, the number of tasks involving animals ( $n = 3, 6.66\%$ ) was one more than the number of tasks involving geography ( $n = 2, 4.44\%$ ), free time activities ( $n = 2, 4.44\%$ ), and recycling ( $n = 2, 4.44\%$ ). See Figure 10 for exemplary tasks involving animals and free time activities contexts.

Chelsea has four hours of free time on Saturday. She would like to spend no more than  $\frac{2}{3}$  of an hour on each activity. How many activities can she do during that time? Justify your procedure.



US2, p. 310

**Example**

5. The average adult male Giant Panda weighs about  $1\frac{1}{5}$  times as much as the average adult female. If the average weight of a male Giant Panda is 330 pounds, how much does the average female Giant Panda weigh?

To find the average weight, solve the equation  $330 \div 1\frac{1}{5} = \square$ .

$$330 \div 1\frac{1}{5} = \frac{330}{1} \div \frac{6}{5}$$

Write the mixed number as an improper fraction.

$$= \frac{330}{1} \times \frac{5}{6}$$

Multiply by the reciprocal.

$$= \frac{\overset{55}{\cancel{330}}}{1} \times \frac{5}{\underset{1}{\cancel{6}}}$$

Divide 330 and 6 by their GCF, 6.

$$= \frac{275}{1} \text{ or } 275$$

Simplify.

So, the average female Giant Panda weighs about 275 pounds.

US2, p. 328

Figure 10. Tasks involving animals and free time activities contexts in US2

See Figure 11 for illustrative tasks involving geography and recycling contexts.

25. Reaner Recycling shreds  $\frac{7}{8}$  ton of aluminum each day. The machines can shred  $\frac{1}{24}$  ton aluminum per cycle. How many cycles will be needed to shred the aluminum?



26. **Reason Abstractly** Reaner Recycling collected  $\frac{7}{4}$  ton of aluminum last Saturday. If  $\frac{7}{8}$  ton of aluminum can be shredded each day, how many days will it take to process what was collected on Saturday?

US2, p. 323

**Extreme Geography** The deepest point in Earth's oceans is the Mariana Trench, which is located  $6\frac{4}{5}$  miles beneath the ocean's surface. The average depth of Earth's oceans is  $2\frac{1}{2}$  miles. By contrast, the highest elevation of Earth is Mt. Everest, which is about  $5\frac{1}{2}$  miles high.

1. Write a division expression to find how many times as deep the Mariana Trench is than the average depth of the ocean.

$\frac{\text{Mariana Trench}}{\text{Average Ocean Depth}} \div \frac{\text{Average Ocean Depth}}{\text{Average Ocean Depth}}$

2. Write a division expression to find how many times as tall Mt. Everest is than the average depth of the ocean.

$\frac{\text{Mount Everest}}{\text{Average Ocean Depth}} \div \frac{\text{Average Ocean Depth}}{\text{Average Ocean Depth}}$

US2, p. 325

Figure 11. Tasks involving recycling and geography in US2

Tasks involving auto accessory and room contexts were existent only in US3. More specifically there were 2 auto accessory tasks and 2 room tasks in this textbook. Tasks that exemplify room and auto accessory contexts are presented in Figure 12.

The larger room is twice as long as the smaller room. How long is the larger room?

If the length of the smaller room is divided into 4 equal parts, how long is each part?

$20\frac{4}{5}$  feet      ?

US3, p.48

Damon has  $37\frac{1}{2}$  inches of space on his car bumper that he wants to use for bumper stickers. How many short bumper stickers can Damon fit side by side on his car bumper?



$37\frac{1}{2}$  inches

**GO GREEN GO** Long = 15 inches

**I ♥ ROBOTS** Medium =  $10\frac{3}{4}$  inches

**MADE IN USA** Short =  $6\frac{1}{4}$  inches

US3, p. 44

Figure 12. Tasks involving rooms and auto accessories in US3

## Discussion and Implications

The purpose of this study was to compare sixth grade mathematics textbooks, three from Turkey and three from the US, in terms of fraction division contexts. The findings showed that TR1, TR2, and TR3 included 33, 18, and 15 fraction division tasks while US1, US2, and US3 included 20, 169, and 130 fraction division tasks, respectively. This uncovered that the number of fraction division tasks included in textbooks varied to a considerable extent both within and across the selected two education systems. More importantly, the average number of fraction division tasks in the US textbooks was found to be far more, nearly five times as many, than that in Turkish textbooks. This finding is consistent with the findings of previous Turkish and the US textbook comparison studies that focused on a particular mathematics topic such as fraction multiplication (e.g., Kar, Güler, Şen & Özdemir, 2018) or on whole topics in one of the middle grades (e.g., Özer & Sezer, 2014). For a textbook, having more number of tasks does not guarantee having more quality and cognitively more demanding tasks, but they offer more alternatives to the mathematics teacher by allowing her/him to choose whatever (s)he wants when exploring these tasks with her/his students (Barcelos Amaral & Hollebrands, 2017). Acknowledging this idea, a number of studies suggested increasing the number of tasks included in Turkish mathematics textbooks (e.g., Aydoğdu İskenderoğlu & Baki, 2011; Özer & Sezer, 2014) to provide more learning opportunities for students to develop their understanding of fraction division.

Cady et al. (2015) argued that “U.S. middle school curricula are highly repetitive, unfocused, unchallenging, and incoherent” (p. 105). Consequently, a great deal of repetition in the US textbooks might have given rise to the unduly high number of fraction division tasks in these textbooks. Indeed, prior research has criticized the US mathematics textbooks for the amount of repetition (e.g., Jones, 2004; Schmidt, McKnight & Raizen, 2007). Meanwhile, Alajmi (2012) conjectured that repetition in the US textbooks might be associated with the introduction of fractions to the students as early as the first grade. Hence, mathematics education community (i.e., research findings related to division of fractions) may guide the Turkish and the US textbook developers in reaching a common ground about the optimum number of fraction division tasks and consequently warrant a more balanced distribution of fraction division tasks across different education systems.

The findings also showed that the rate of contextual tasks also varied to a considerable extent both within and across the selected mathematics textbooks. More particularly, TR1, TR2, and TR3 included 27 (81.81%), 10 (55.55%), and 4 (26.66%) contextual tasks, while US1, US2, and US3 included 7 (35.00%), 45 (26.62%), and 35 (26.92) contextual tasks, respectively. Overall, more than half of tasks (62.12%) included in Turkish textbooks were contextual while about only a quarter of US textbook tasks (27.28%) were contextual. Prior research also revealed that the majority of tasks included in the US textbooks were non-contextual (e.g., Barcelos Amaral & Hollebrands, 2017; Cady et al., 2015; İncikabı & Tjoe, 2013; Kar et al., 2018; Li, 2000; Özer & Sezer, 2014; Son & Senk, 2010; Yang, Tseng & Wang, 2017). Contextual features of textbooks have impact on students’ proficiency in solving mathematical tasks (Stigler et al., 1986). Besides, learning fraction division conceptually entails using contextual tasks (Li, 2008). Thus, it can be argued that Turkish and the US students using the mathematics textbooks selected in this study may not be provided with equal opportunities to work on contextual fraction division tasks. The lack of opportunities to learn in mathematics textbooks may lead to difficulties in solving context-based tasks involving division of fractions (Wijaya et al., 2015b). Increasing use of tasks that are set in a real life context in mathematics classrooms, may give rise to an atmosphere which facilitates higher level understanding (Gu et al., 2004). Non-contextual fraction division tasks, on the other hand, may be important in terms of helping retention of the division algorithm and developing fluency with it. However, predominant use of non-contextual tasks in the classroom may foster procedural understanding, whereas it may hinder students’ development of conceptual understanding of fraction division. Hence, the US textbooks may pay more attention to including contextual fraction division tasks to provide students more

opportunities to employ the knowledge of the context, choose appropriate information to understand conceptually, and work on such tasks.

In this study, considerable variations were also found both within and across the selected mathematics textbooks with respect to the type of tasks presented. Overall, Turkish textbooks included eight different contextual tasks, while the US textbooks included sixteen different contextual tasks regarding division of fractions. This showed that the US textbooks were richer than the Turkish textbooks in terms of variety of contextual tasks. Foods and beverages, handicrafts, travelling, carpentry, strips, gardening, and household goods contexts were existent in both countries' textbooks, while shopping context was existent only in one Turkish textbook (i.e., TR1). Student dependent contexts, sports, land, painting, animals, geography, free time activities, recycling, auto accessories, and room contexts were existent only in the US textbooks. In particular, all textbooks paid attention to including tasks related to foods and beverages context. This finding is in line with the findings of Cady et al. (2015), which focused on exploring the similarities and differences existing in three US middle school mathematics textbooks (i.e., CMP, Thematics, and Glencoe) in terms of presentation of fractions. Textbooks' frequent use of tasks involving food and beverages context may be explained via Barcelos Amaral and Hollebrands' (2017) tripartite categorization of context-based tasks as "problems most students can relate to and make sense of, problems students with particular experiences can relate to, and problems that have contexts that students are not likely to encounter in their everyday lives" (p. 9). Simply put, it is possible to say that all students may have some experiences with foods and beverages and thus they can easily make sense of tasks involving such contexts.

Mathematics education researchers and practitioners have lately placed strong emphasis on incorporating problem posing into school mathematics instruction (e.g. Cai, Hwang, Jiang & Silber, 2015; Singer, Ellerton & Cai, 2013). Besides, MoNE (2013) considers problem posing as one of the most crucial mathematical process skills. Despite the centrality of problem posing in mathematics education, fraction division tasks involving student dependent contexts (i.e., problem posing tasks) were absent from Turkish textbooks. Problem posing tasks may foster students' conceptual understanding, and improve their mathematical reasoning and communication ability (Cai et al., 2015). Thus, Turkish mathematics textbook developers are suggested to pay attention to and incorporate student dependent contexts into fraction division tasks in future preparation of school textbooks.

This study explored the characteristics of textbooks only in terms of type and frequency of context-based fraction division tasks. Therefore, there are some suggestions for future research. First, researchers may examine other characteristics of textbooks such as cognitive demand levels of fraction division tasks, type of information provided by the tasks (i.e., matching, missing, and superfluous information), and so forth. Second, the textbooks were selected from the US and Turkey. These two countries had lower levels of mathematics performance in PISA 2015 (OECD, 2016). The mean mathematics score of the fifteen-year-old US and Turkish students were statistically significantly below the OECD average. Thus, researchers may compare fraction division task contexts included in Turkish or US mathematics textbooks with the ones from high-achieving Asian countries such as Singapore, Hong Kong, China, and Japan. Such comparisons may allow for more quality and rich contextual tasks for Turkish or US students.

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#### **Appendix: Textbooks analyzed**

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