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The Effect of Pattern-Based Mathematics Education Program on 61-72-Month-Old Children's Reasoning Skills*

Research Article

Feride GOK COLAK[1], Adalet KANDIR[2]

[1] *Yıldız Technical University, Faculty of Education, Department of Elemantary Education, Istanbul, Turkey,* ORCID: 0000-0001-5196-9178
[2] *Gazi University, Faculty of Education , Department of Elemantary Education, Ankara, Turkey,* ORCID: 0000-0002-9917-2587

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ABSTRACT

This study was carried out to investigate the effect of Pattern-based Mathematics Education Program (PMEP) on the reasoning skills of 61-72-month-old children. Mixed design was used in this study. As the mixed design method is experimental, pretest / posttest experimental design with control group is used. The study group was comprised of a total of 40 children (61-72 months); 20 in the experimental group and 20 in the control group. The qualitative part of the study consists of observations and interviews with teachers. General Information Form, Cognitive Abilities Test Form-6 (CogAT-6), Teacher Observation Form, Teacher Interview Form and Parent Involvement Evaluation Form were used as data collection tools. A total of eight weeks 'Pattern-based Mathematics Education Program' (PMEP) was applied to the children in the experimental group five days a week. Cognitive Abilities Test Form-6 was given to the experimental and control groups as pretest and posttest. The same test was applied to the experimental group as retention test three weeks after the posttest. A special statistical package program was used to interpret the quantitative analysis of the data. As a result of the study, there was a significant difference between the Verbal, Numerical and Nonverbal dimension pretest and posttest scores of the experimental and control group children in favor of the experimental group (p <.05). It was determined that the difference between the Verbal and Nonverbal posttest and retention test scores of the children in the experimental group was significant, whereas the difference between posttest and retention scores in numerical dimension was not significant (p> 0.05). As a result of the analysis of the qualitative data, it was observed that the teachers were successful in the implementation of the Pattern-based Mathematics Education Program. In addition, it was observed that the education program contributed to the knowledge of teachers. When teachers' opinions were analyzed, it was determined that the education program process was generally positive, and they gained deeper knowledge about the concept of pattern. In addition, it has been suggested that the education program should be adapted to younger ages for long-term implementation.

Keywords:
early childhood education, mathematics education program, pattern, reasoning

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[1] Corresponding author's address: Yildiz Technical University, Faculty of Education, Department of Elemantary Education
e-mail: fgok@yildiz.edu.tr
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Introduction

Research has shown that many concepts related to mathematics have taken place in our lives since birth (Brewer, 2001; Brown, Molfese and Molfese, 2008; Clements and Sarama, 2007). One of these concepts is the pattern. With birth, babies begin to define the pattern of the human face (Murray and Andrews, 2000) and meet with the world, which is a set of patterns. The infant brain, which defines the pattern of the human face, is so surprising that it cannot be underestimated. Thus, Pound (2006) stated that the human brain is successful in defining patterns. In addition, patterns help children to discover mathematical concepts (Tanışlı, 2008). According to the literature, it is seen that there are many definitions related to the concept of pattern (Birken and Coon, 2008; Souviney, 1994; Papic and Mulligan, 2005, p. 609; Mulligan and Mitchelmore, 2013). The pattern is defined as the situation in which the relational numbers, spaces, or measurement variables have predictable regularity (Mulligan and Mitchelmore, 2013) in a course of regular development of events or objects (Turkish Language Association (TDK), 2013). The discovery of the pattern is a process. This process consists of four stages. Pattern discovery process is as follows: recognizing the pattern, defining the pattern, continuing the pattern and producing a new pattern (Akman, 2010, p. 132).

In daily life, various types and shapes of patterns are encountered. According to the literature, it is seen that different categories have been formed about the types of patterns. Smith (2006) states that the pattern he describes as patterning in early childhood has evolved in three different ways: repeating pattern, expanding pattern and relationship pattern. According to Papic (2007), there are three different types of repeating patterns: Linear Pattern or Tower Pattern, Circle Pattern and Hopscotch pattern.

• Linear Pattern or Tower Pattern: The linear pattern is defined as a pattern of repeating patterns that are vertical or horizontal.

• Circle Pattern: A repeating pattern format that has a certain shape (square rectangle or circle, etc.) with the same starting and ending points.

• Hopscotch pattern: It is a repeating pattern form created as a horizontal arrangement of some of the pattern elements and some of them vertically.

Changing patterns are a regular ordering of a set of numbers, shapes or concrete materials (Billstein, Libeskind and Lott, 2004, p. 5). The relationship between terms in the changing patterns follows an expanding or narrowing course. The changing patterns can be grouped in three different ways as constant changing, increasing and decreasing (Olkun and Yeşildere, 2007, p. 13). In addition to these patterns, relational patterns and musical patterns are also found in the literature. Relational patterns are patterns that do not exhibit constant or incremental variability but vary in order. Such patterns are used during the transition to function and provide functional thinking (Uygur-Kabael and Tanışlı, 2010). Music patterns can be defined as the process of discovering patterns in the auditory sense. There are musical patterns formed depending on the successive sound or words (Geist, Geist and Kuznik, 2012).

The importance of the patterns encountered in various types and shapes in daily life in terms of children's cognitive development cannot be denied. According to Devlin (2000), mathematics is the science of pattern. Even the power of mathematics lies in relationships and transformations leading to patterns and generalizations (Warren, 2005, p. 305). The pattern recognition ability is therefore the development of the mathematical thinking of the human brain and should be used to enable the young children to benefit mathematically in the classroom (Pound, 2006). Moreover, the share of patterns in the discovery and understanding of mathematical concepts is large (Burns, 2000 p.112, Charleswort and Lind, 2010 p. 214; Palabıyık and Akkuş-İspir, 2011; Tanışlı, 2008). Patterns contain matching, sorting, grouping and comparison skills, and thus establish a relationship between mathematical concepts (Palabıyık and Akkuş-İspir, 2011). Patterns also play an important role in the abstraction of mathematical thoughts and relationships, and in the generalization of relations (Papic and Mulligan, 2005, p. 609). However, the pattern is also of great importance for the development of reasoning skills in early childhood. (English, 2004; Mulligan, Prescott and Mitchelmore, 2004; Waters, 2004).

The concept of "reasoning" is defined in different ways in the literature (İnal, 2011; Peresini and Webb, 1999; Turkish Language Association (TDK, 2013); Umay, 2003). Reasoning, a way out to solve a problem (TDK, 2013), is an activity that involves different kinds of thinking (Peresini & Webb, 1999). In other words, the reasoning ability, which is one of the cognitive skills, can be explained as a new decision-making process by creating new implications with the help of current knowledge about a new situation (event or topic) (İnal, 2011). There are many different sub-categories of reasoning. Some of these are analogical reasoning, algebraic reasoning and verbal reasoning. English (2004, p. 2) defines the concept of analytic reasoning as "reasoning ability by using relational patterns". According to Van de Walle, Karp and Williams (2012), Algebraic Reasoning involves regularity and pattern recognition, generalization and shaping in all aspects of mathematics. Algebra is a tool for explaining the changes in the universe and the relationships between objects or events (OECD, 2009). Steele (2005) expresses the concept of "Algebraic Thinking" as the ability to identify, analyze and generalize patterns. In the definition of algebra and algebraic thinking concepts, the concept of "pattern" is noteworthy. When all of these definitions are taken into consideration, it is observed that the patterns are used in the process of reasoning. Learning the concept of pattern is a prerequisite for algebraic reasoning. Although algebra teaching is considered suitable for middle school or high school students, it can be encouraged to use algebraic reasoning with number-processing pattern studies in younger students (NCTM, 2000). As a matter of fact, verbal reasoning ability in the literature is defined as the ability to interpret and evaluate the concepts and to draw conclusions. (Lohman and Hagen, 2003, p. 7). The ability of verbal reasoning to solve verbal problems, to see the relations between words and sentences, to find the logical relationship between words. In order to support verbal reasoning skills, it is necessary to support the ability to establish cause-effect relationship, to make sense of rules and principles, to classify concepts, objects or events (Van der Sluis, De Jong and Van der Leij, 2007, pp. 429-430). It also emphasizes that such reasoning should involve exploring the patterns and identifying the repeating unit again (Pilten, 2008, p. 46).

In the systematics of the pattern, there are skills such as resembling similarities or differences, establishing relationships between elements, matching, grouping. In addition, in order to generalize the pattern, it is necessary to make an estimation and evaluation. Reasoning ability is also a process that requires establishing a cause-and-effect relationship. Therefore, it is thought that pattern-based activities support reasoning skills. In recent years, importance given to the development of reasoning skills is increasing both in Turkey and around the world. Instead of memorizing mathematical concepts of children, thinking, questioning, reasoning has come to the forefront. It has been reported that it is necessary to use patterns in the National Education Programs to improve the reasoning ability (MEB, 2013).

When the studies about reasoning are examined, it is seen that there are many studies on children but limited number. When these studies are examined, it is revealed that they have aimed at the examination of the reasoning and proofing skills of the children at different levels of education (Altıparmak and Öziş, 2005), the examination of the reasoning of the six-year-old children according to the situations faced by them (Başara-Baydilek, 2010), the relationship of spatial reasoning in mathematics learning (Mulligan, 2015), examining the relationship between rhytmic perceptions and reasoning (Aydın and Mertoğlu, 2006). In addition, the Cognitive Assumption Test (Lohman and Hagen, 2003) and Early Mathematical Reasoning Skills Assessment Tool (Ergül, 2014) suggest that assessment tools for reasoning skills have been developed. In particular, there were no studies examining the relationship between reasoning skills and early childhood literature in the literature.

However, it is seen that the studies based on the pattern are mostly directed to the students in the middle and high school levels when the literature review is carried out for domestic studies (Aslan, 2011; Bursalıoğlu, 2010; Kutluk, 2011; Özdemir, 2013; Palabıyık, 2010; Tanışlı, 2008; Yaman, 2010; Yakut-Çayır, 2013; Yeşildere and Akkoç, 2011). However, when the international field is examined in the literature, patterns have been generally used to prepare the foundations of mathematical thinking in the early childhood period and also the patterns are used for the construction of algebra (Clements and Sarama, 2007; Lee, Ng, Bull, Pe and Ho 2011; Mulligan, Mitchelmore, Kemp, Marston and Highfield, 2008; Papic, 2015). Therefore, the main purpose of this study is to examine the effect of the Pattern-based Mathematics Education Program on the reasoning skills of 61-72 months old children. The following questions are sought for this basic purpose:

1. Is there a significant difference between the Reasoning Skills pre-test scores of the experimental and control group children?

2. Is there a significant difference between the Reasoning Skills pre-test / post-test score scores of children in the experimental group?

3. Is there a significant difference between the Reasoning Skills pre-test / post-test scores of children in the Control Group?

4. Is there a significant difference in the Reasoning Skills post-test scores of the experimental and control group children?

5. Is there a significant difference between the post-test and retention test scores of the Reasoning Skills of children in the experimental group?

6. What are the teachers' views on the Pattern-based Mathematics Education Program?

7. What are the opinions of families about the Pattern-based Mathematics Education Program's family participation activities?

Methodology

As both qualitative and quantitative data were used in the study, mixed design method was used. In the mixed design method, there is a study that combines qualitative and quantitative methods together with the strength of the complementary force and its strengths (Johnson and Christensen, 2014, p. 51). Therefore, pretest / posttest, experimental design with control group was used in the quantitative part of the study. In the qualitative part of the study, the case study was used. In addition, triangulation as the reason for using the mixed design can be shown as the attempt to bring together the results of different methods in a way that supports each other (Johnson and Christensen, 2014, p. 439).

Study Group

In the study, multi-level sequential sampling was used from mixed sampling patterns. The reason for the multi-level successive sample selection is to benefit from children in the quantitative part of the study before and after education, in the qualitative research part of the study, and finally from the teachers and the parents of the children in the experimental group (Johnson and Christensen, 2014, p. 238).

In the quantitative part of the study, the study group consisted of 61-72-month-old 40 children (6 females and 14 males in the control while 11 females and 9 males in the experimental group) attending to the kindergarten affiliated to a public university located in İstanbul/Turkey during 2015-2016 academic year. The study group in the qualitative part consists of the parents of the children in the experimental group and two class teachers. While the experimental and control groups were formed in the quantitative part of the study, it was paid attention to the normal development of the children in both groups and not having participated in any other mathematics education program before. In addition, care was taken to ensure that children included in the study group were not included in a special education program on any subject other than the Ministry of National Education (MoNe) Preschool Education Program. Accordingly, two different preschool education centers were selected for the experimental group on the campus of a public university by appropriate sampling method. A total of four classes were included in the study, two for the experimental group and two for the control group.

Data Collection

As data collection tools, "General Information Form" including the items regarding the demographic characteristics of the children and parents involved in the research; "Teacher Observation Form" to evaluate the performance of the teacher in the education process; "Teacher Interview Form" to collect teachers' views about the educational process; "Parent Involvement Evaluation Form" were all used. In order to evaluate children's reasoning skills, five different data collection tools were used including the Cognitive Abilities Test Form-6 (CogAT Form 6) developed by Lohman and Hagen (2001).

• *The General Information Form* was prepared by the researchers to gather information about the parents and the children included in the research. The General Information Form includes items regarding the demographic information such as gender, age, number of siblings, birth order, pre-school education process, education level of the parents and their professions and ages.

• *Teacher Observation Form* was prepared by the researchers to evaluate the teacher's performance in the implementation process of the Pattern-based Mathematics Education Program.

• *Teacher Interview Form* is composed of semi-structured questions for teacher views after the implementation of the Pattern-based Mathematics Education Program.

• *Parent Involvement Evaluation Form* contains evaluation questions for parent involvement activities prepared for the purpose of ensuring the permanence of activities carried out at school with the children and prepared by the researchers. In the form, questions about with whom the activity was done, how much time was allocated to the activity, the status of completion of the activity, the difficulties experienced by the child during the implementation of the activity, whether or not the activity was carried out in accordance with the stages of the activity, the effectiveness of the activity in the development of the child.

• *Cognitive Abilities Test Form-6 (CogAT Form-6)* which was developed by Lohman and Hagen (2001) in the United States and adapted to Turkish by İnal and Ömeroğlu (2011) was used. CogAT Form 6; using verbal, numerical and nonverbal skills, aims to evaluate the level of access of children in reasoning skills (Alp and Diri, 2003, p. 21; İnal and Ömeroğlu, 2011; Lohman and Hagen, 2003, p.1). Total KR-20 reliability coefficient of CogAT Form 6 is 0.91. The verbal dimension KR-20 reliability coefficient is 0.76, the numerical dimension KR-20 reliability coefficient is 0.82 and the non-verbal dimension KR-20 reliability coefficient is 0.70 (İnal, 2011).

Data Collection Process

Before starting the study, the relevant written consents were obtained from the parents for the study and approval letter was received from the university ensuring that the procedure does not violate the ethical rules of the scale. In addition to these permits, "Consent Form of the Educator" was signed by the teachers stating that they participated the study on their own will and also the school directors provided written consent for the study to be carried out.

Quantitative data were collected by Cog-AT Form-6 (İnal and Ömeroğlu, 2011). Qualitative data were obtained from simultaneous observation and interview forms. Then, using these findings, it was examined whether the data support each other.

In addition to the data collection tools mentioned above, a total of 40 activities (8 contingency activities) and a total of eight family participation studies sent to families on a Friday basis were carried out by the Pattern-based Mathematics Education Program (PMEP).

Pattern-based Mathematics Education Program (PMEP), which was developed by researchers, is based on the development of awareness of 61-72-month old children. This program aims to contribute to the mathematical thinking skills of children by supporting patterning skills, including the ability to discover, recognize, copy, find the pattern unit, continue the pattern, and create an original pattern. Pattern-based Mathematics Education Program, which combines active participation with children's interest and curiosity, combines life, play, and mathematics. The Pattern-based Mathematics Education Program, which allows children to explore patterns, repeating key areas, changing and relational patterns, is a training program that creates various learning experiences with different forms of activities.

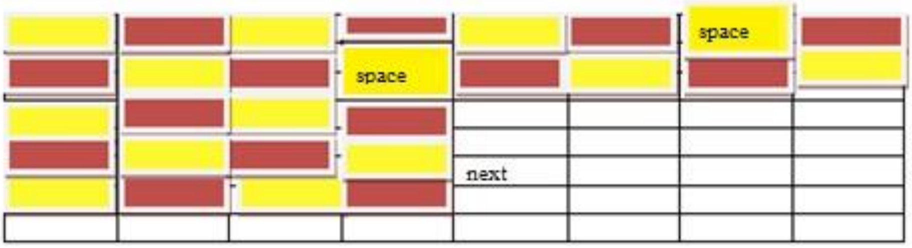
The activities aimed at the development of repetitive, changing and relational patterns in the Pattern-based Mathematics Education Program were prepared according to the principles such as from simple to complex, from concrete to abstract and readiness / maturity in terms of sound, form, number and movement. In addition, this program integrates with the Turkish, music, play, science, movement, drama, reading, writing and art activities stated in MEB Preschool Education Program (2013). In this context, the Pattern-based Mathematics Education Program aims to raise awareness of pattern and mathematics that exist in daily life and which are also related to other disciplines.

Pattern is directly related to all sub-branches of mathematics such as number-processing, algebra, geometry and probability. They are intertwined with "Number and the Process" sub-branch in terms of the discovery of the patterns within the numbers and with respect to the patterns formed from the numbers; "Algebra" in terms of the letter expressions, the functions of the abstraction and transformation through the patterns; "Geometry" with respect to the discovery, the continuation and production of shape patterns; and "Probability" in terms of the possibility of creating a pattern by considering all possibilities can be specified. For this reason, it is thought that the Pattern-based Mathematics Education Program will have all-round mathematical knowledge and skills to include all sub-branches of mathematics.

The Pattern-based Mathematics Education Program is a set of activities planned to enable the creation of appropriate conditions for the development of attention, perception, memory, intelligence, reasoning and problem-solving elements of cognitive ability. Pattern-based Mathematics Education Program has a structure that supports the development of memory by creating similar patterns or unique patterns from its memory, which draws the attention of children with unrecognized pattern examples in daily life, develops different areas of intelligence with patterns in different forms of presentation.

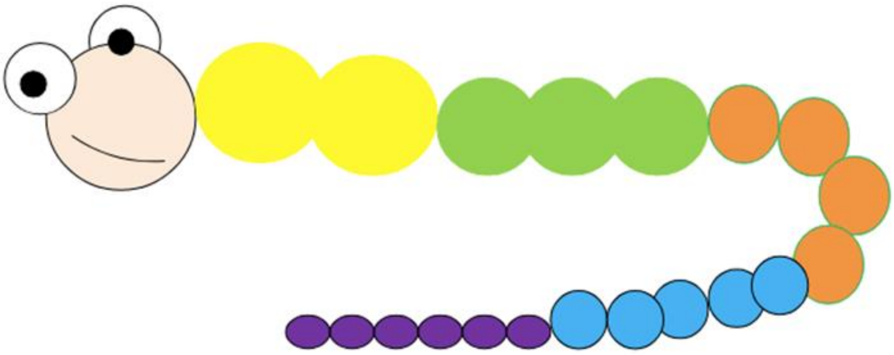
The philosophy, principles and characteristics of the Pattern-based Mathematics Education Program prepared for the purpose of the study were determined by examining the studies, projects applied and scientific publications in the related literature. Then, the Pattern-based Mathematics Education Program was prepared to be integrated into the MEB Preschool Education Program (2013) for 61-72-month-old children. The activities, materials, methods and techniques of the Pattern-based Mathematics Education Program were created by the researcher. With the validity of the Pattern-based Mathematics Education Program, a total of 40 activities (8 contingency activities) and 8 parent involvement activities were implemented for a total of 20-30 minutes per day for five days and eight weeks.

Examples of repeating, changing and relationship patterns are given below. In repeating pattern activity, children are divided into small groups of 4 people. Then each group is distributed to a box. On the side faces of the box, there is Visual 1. Children are asked to paint the colors that will come into the spaces with finger paint and to continue the pattern.



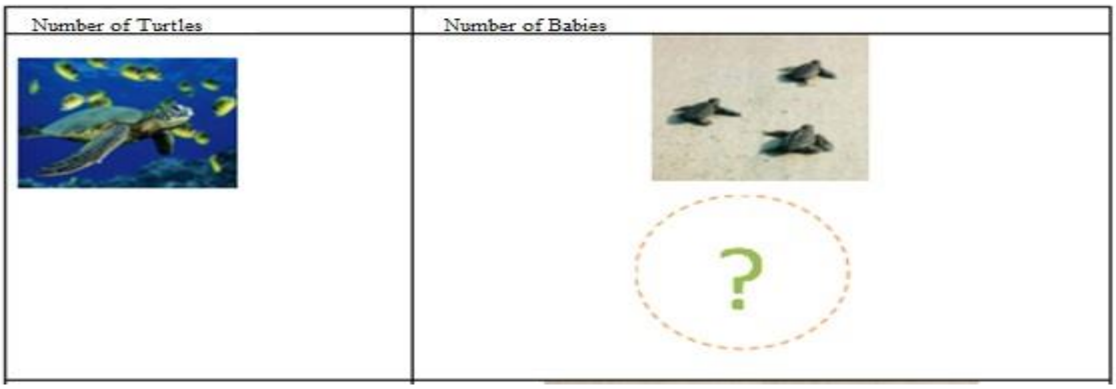
Visual 1. An example of a repeating pattern

In the changing pattern activity, the worm's figure from the Visual-2 is removed from a cloth bag and the story of the worm is read. Each color of the worm is adapted to the story and children are asked to discover the rule in their color. Children are then provided with material to make their own worms both in two dimensions and in three dimensions. Children are expected to create similar patterns.



Visual 2. An example of a changing pattern

In the relationship pattern activity, children are introduced to a toy similar to a turtle in Visual-3. The turtle is said to have babies. A deep box filled with kinesthetic sand is said to hide the eggs. Step by step the eggs are reached with the children. There are 3 eggs in each hidden section. Children are asked questions about the rule. Activity paper in Visual 3 is then distributed, where children are asked to establish a relationship with the number of babies against the number of turtles. The relational pattern is cut after a few steps as it is difficult for children to perceive from a developmental perspective.



Visual 3. An example of relationship pattern

During the implementation of the Pattern-based Mathematics Education Program, one of the researchers observed the experimental group using the participatory observation method. The researcher was a direct participant in the experimental group during the implementation of the Pattern-based Mathematics Education Program. While observing, the application of the program was recorded with the camera and the images were revisited by the researcher in accordance with the characteristics of the Teacher Observation Form.

In addition, the researcher sent parent involvement evaluation forms to the parents of the children in the experimental group during the implementation (eight times in total) and interviewed the teachers after the implementation of the Pattern-based Mathematics Education Program was completed. Standardized open-ended interview method was used in the interviews. The findings were interpreted by descriptive analysis.

The researcher gave detailed information about the characteristics of the program to be applied, the methods and techniques to be used, which methods and techniques will be used, the classroom environment and the relationship with the children to be established in the experimental group related to the Pattern-based Mathematics Education Program. The Pattern-based Mathematics Education Program was implemented for a total of eight weeks, five days a week. Camera recording and photographing of the activities in the experimental group were done. In the Pattern-based Mathematics Education Program implemented by the teacher under the supervision of the researcher, the same situation was repeated for each week for eight weeks. In the control group, the program was implemented by the classroom teachers in the natural flow by continuing the implementation of the current educational program. In addition, the methods, techniques and materials used in the experimental group were not used in the control group. However, the parents of the children in the control group we informed that they could be educated at the end of the research if they wish.

Data Analysis

The quantitative data collected for the purpose of the study was evaluated and statistical analyzes were made. The data collected for the 61-72-month-old preschoolers through the Cognitive Abilities Test Form-6 and the General Information Form were analyzed by means of the statistical package program. Shapiro Wilk's was used because the number of groups was smaller than 50 when investigating variables from normal distribution (Büyüköztürk, 2016). When examining the differences between the groups, Mann Whitney U Test was used because the variables did not come from the normal distribution. Since the number of units is more than 20, standardized z values are given for Mann Whitney U Test. The Wilcoxon Signed Rank Test was used because the variables did not come from the normal distribution when examining the difference between two dependent variables. The qualitative data collected from parents and teachers for the purpose of the research were interpreted by descriptive analysis.

Findings

In order to investigate the effect of the Pattern Based Mathematics Education Program on the reasoning skills of 61-72 months old children, the findings of the research were examined in two parts.

1. **Comparison of Cognitive Abilities Test Form-6 pretest, posttest, pretest / posttest scores of children in the experimental and control group, and the comparison of posttest / retention test scores of children in the experimental group**

Table 1. Cognitive Ability Test Form-6 Pretest Mean Scores of Children in the Experimental and Control Group and Descriptive Statistics

Pretests		Group						Mann Whitney U Test		
		n	$\overline{X}$	Med.	Min.	Max	Sd	Mean Rank	z	p
Vocabulary	Control	20	13.7	14	8	17	2.66	19.63	-0.479	0.632
	Experiment	20	14.4	14	11	18	1.93	21.38		

	Total	40	14.05	14	8	18	2.32			
Verbal Reasoning	**Control**	20	12.3	13	5	17	3.15	19.25	-0.682	0.495
	Experiment	20	12.9	13	7	17	2.27	21.75		
	Total	40	12.6	13	5	17	2.73			
Verbal Dimension	**Control**	20	26	26	13	34	5.23	19.28	-0.666	0.505
	Experiment	20	27.3	27	18	33	3.73	21.73		
	Total	40	26.65	26.5	13	34	4.53			
Relational Concepts	**Control**	20	10.85	11	6	18	3.28	18.83	-0.912	0.362
	Experiment	20	11.6	11.5	7	17	2.56	22.18		
	Total	40	11.23	11	6	18	2.93			
Quantitative Concepts	**Control**	20	12.6	12	7	18	3.39	16.35	-1.72	0.052
	Experiment	20	15	15	6	20	3.34	24.65		
	Total	40	13.8	14	6	20	3.54			
Numerical Dimension	**Control**	20	23.45	22	14	34	5.53	16.9	-1.955	0.051
	Experiment	20	26.6	26	18	37	4.97	24.1		
	Total	40	25.02	23.5	14	37	5.43			
Shape Classification	**Control**	20	11	10.5	4	19	4.38	20.45	-0.027	0.978
	Experiment	20	10.85	10	6	15	2.66	20.55		
	Total	40	10.93	10	4	19	3.58			
Matrices	**Control**	20	12.75	12.5	3	18	3.49	24.13	-1.983	0.057
	Experiment	20	10.7	12	4	15	2.98	16.88		
	Total	40	11.72	12	3	18	3.37			
Non-verbal	**Control**	20	23.75	23	15	36	6.4	22.25	-0.949	0.343
	Experiment	20	21.55	22.5	13	27	4.43	18.75		
	Total	40	22.65	22.5	13	36	5.54			

According to Table 1, it was seen that the mean scores of the verbal dimension pretest scores of the children in the experimental group were 27.3 and the mean score of the control group was 26. The mean scores of the pretest scores of the children in the experimental group were 26.6 and 23.45 in the control group. The mean scores of non-verbal dimension pretest scores of the children in the experimental group were 21.55 and the control group mean scores were 23.75. There was no significant difference between the groups in terms of pretest scores ($p>0.05$). In all dimensions and sub-dimensions, it can be said that both groups are equivalent in terms of pretest.

Table 2. Cognitive Abilities Test Form 6 Pretest / Posttest Descriptive Statistics of Experimental Group Children and Wilcoxon Signed Rank Test Results

							Wilcoxon Signed Rank Test		
Experimental Group	**n**	$\overline{X}$	**Med.**	**Min.**	**Max.**	**sd**	**Mean Rank**	**z**	**P**
Verbal Dimension Pretest	20	27.3	27	18	33	3.73	5.5	-3.397	**0.001***
Verbal Dimension Posttest	20	30.55	31.5	24	35	3.24	10.53		

Numerical Dimension Pretest	20	26.6	26	18	37	4.97	2.83	-3.358	**0.001***
Numerical Dimension Posttest	20	31.6	31.5	16	40	6.09	10.83		
Non-verbal Dimension Pretest	20	21.55	22.5	13	27	4.43	5	-3.231	**0.001***
Non-verbal Dimension Posttest	20	25.5	25	19	34	4.03	10.94		

$p<0.05^*$

According to Table 2, Verbal Dimension pretest mean score of the children in the experimental group was 27.3 and the posttest score was 30.55. The mean scores of the pretest scores were 26.6, the posttest mean score was 31.6 and the non-verbal dimension was 21.55 and the post-test mean score was 25.5. When the table is examined carefully, there is a statistically significant difference in all dimensions of the children in the experimental group according to the results of Wilcoxon Signed Rank Test which shows a significant difference between the pretest and posttest scores of the children in the experimental group (p <0.05). The verbal, numerical dimension and non-numerical dimension posttest score of the children in the experimental group were significantly higher than the pretest scores. In line with these results, it was found out that the observed change in the scores of children in the experimental group who participated in the Pattern-based Mathematics Education Program was in favor of the experimental group compared to the control group children.

Table 3. Descriptive Statistics Regarding Verbal Dimension Pretest / Posttest Scores of Control Group Children and Wilcoxon Signed Rank Test Results

							Wilcoxon Signed Rank		
Control Group	**n**	$\overline{X}$	**Med.**	**Min.**	**Max.**	**Sd**	**Mean Rank**	**z**	**P**
Verbal Dimension Pretest	20	26	26	13	34	5.23	10.83	-2.313	**0.021***
Verbal Dimension Posttest	20	29	29	19	35	4.1	9.23		
Numerical Dimension Pretest	20	23.45	22	14	34	5.53	8.81	-0.99	0.322
Numerical Dimension Posttest	20	24.85	24	14	33	5.58	10.86		
Non-verbal Dimension Pretest	20	23.75	23	15	36	6.4	9.6	-0.04	0.968
Non-verbal Dimension Posttest	20	23.95	23.5	14	38	6.56	10.44		

$p<0.05^*$

According to Table 3, the mean score of the Verbal Dimension pretest scores of the children in the control group was 26 and the posttest mean score was 29. The mean score of the Numerical Dimension pretest was 23.45, posttest mean score was 24.85, the mean of the non-verbal dimension was 23.75 and the posttest mean score was 23.95. When the table was examined, there was no significant difference between the Numerical and Non-verbal Dimension pretest / posttest mean scores of the children in the control group (p> 0.05), while in terms of Verbal Dimension there was a significant difference between the pretest and posttest mean scores (p<0.05).

Table 4. Mann Whitney U Test Results Regarding the Difference Among Cognitive Abilities Test Form-6 Posttest Mean Scores and Standard Deviation of Control and Experimental Group Children

					Mann Whitney U Test		
		n	$\overline{X}$	**Sd**	**Mean**	**Z**	**P**
Verbal Dimension Posttest	Experiment Group	20	30.55	3.24	22.58	1.13	0.259
	Control Group	20	29	4.1	18.43		
	Total	40	29.78	3.73			
Numerical Dimension Posttest	Experiment Group	20	31.6	6.09	26.33	3.158	**0.002***
	Control Group	20	24.85	5.58	14.68		
	Total	40	28.23	6.7			
Non-verbal Dimension Posttest	Experiment Group	20	23.95	6.56	19.08	0.774	0.439
	Control Group	20	25.5	4.03	21.93		
	Total	40	24.72	5.43			

$p<0.05^*$

According to Table 4, it was observed that the verbal dimension posttest score averages of the Cognitive Abilities Test Form-6 of the children in the experimental group were 30.55, and the posttest mean scores of the children in the control group were 29. Numerical Dimension mean scores of the children in the experimental group were 31.6, the posttest mean score of the children in the control group was 24.85, the mean score of the Non-verbal Dimension was 25.5, and the mean score of the children in the control group was 23.95.

When the table is examined carefully, it is seen that there is a significant difference in the Cognitive Abilities Test Form-6 posttest scores of the of the children in the experimental and control groups ($p<0.05$). There was no significant difference between the Verbal Dimension and Non-verbal Dimension posttest scores of control and experimental group children ($p>0.05$). In the experimental group, the Verbal Dimension and Non-Verbal Dimension posttest scores were higher than the control group although the difference was not significant.

Table 5. Mean, Standard Deviation and Wilcoxon Signed Rank Test Results Regarding the Difference Between Verbal Dimension and Sub-dimensions Posttest and Retention Test Scores of Control Group Children

				Wilcoxon Signed Rank		
Experiment Group	**n**	$\overline{X}$	**Sd**	**Mean Rank**	**Z**	**P**
Verbal Dimension Posttest	20	30.55	3.24	3.88	-2.898	**0.004***
Verbal Dimension Retention Test	20	33.4	3.1	10.58		
Numerical Dimension Posttest	20	31.6	6.09	7.08	-1.878	0.06
Numerical Dimension Retention Test	20	33.6	5	10.71		
Non-verbal Dimension Posttest	20	25.5	4.03	6.5	-2.958	**0.003***
Non-verbal Dimension Posttest	20	30.4	6.32	11.5		

$p<0.05^*$

According to Table 5, Cognitive Abilities Test Form-6 Verbal Dimension posttest mean scores of the of the children in the experimental group was 30.55 and the retention test mean score was 33.4. Numerical Dimension posttest mean score was 31.6, the mean of retention test was 33.6 and the Non-verbal Dimension posttest mean score was 25.5 and the mean of retention test was 30.4.

There was a statistically significant difference between the Verbal Dimension and Non-Verbal Dimension posttest and retention test scores of the children in the experimental group ($p < 0.05$). According to these results, the retention test mean scores which were obtained three weeks after the posttest administered at the end of the implementation of the Pattern-based Mathematics Training Program were higher than the posttest mean scores. This situation can be interpreted that the effect of the education program continues in the experimental group.

2. Qualitative data regarding the implementation and evaluation of Pattern-based Mathematics Education Program

In the process of Pattern-based Mathematics Education Program (PMEP), the qualitative data were obtained separately from the parents and classroom teachers of the children in the experimental group. The evaluation form for parent involvement activities in the program was sent to families each week. Data from standardized open-ended questions were digitized in frequency and percentage distributions. In the light of data from the parents, the time allocated to parent involvement activities is approximately 10-15 minutes. (15 (75%) for Activity 1; 18 (90%) for Activity 2, 16 (80%) for Activity 3, 5 (25%) for Activity 4, for 12 (60%) for Activity 5, 4 (20%) for Activity 6, 3 (15%) for Activity 7 and 7 (35%) for Activity 8). As the reason for the increase in the activity periods over time, the activities that are progressing in the training program may require more knowledge and skills and therefore the possibility of parents being forced. In addition, the fact that some parents left the evaluation forms blank can be cited as a reason. It was stated that only the Activity-7 (4 parents) were not interesting among the activities. In the case of performing according to the explanations, they stated that 55% to 95% of the parents performed the activities according to the explanations (95% for Activity 1, 2 and 3, 55% for Activity-4, 80% for Activity-5, 70% for Activity 6, 60% for Activity 7-8). When the difficulty in the implementation process was examined, the majority of the parents did not have any difficulties in the activities, it was determined that only four parents had difficulties in Activity 4, one in the other activities (Activity 1, 2, 5 and 6) or two parents (Activity 7-8) had difficulty. It can be argued that Activity 4 is an activity that requires knowledge and skills and it forces parents in terms of both time and applicability. When the parents' opinions of the parent involvement activities on the development of children were examined, they all stated that the Pattern-based Mathematics Education Program contributed to the development of their children. In the light of all these findings, it can be stated that the family participation activities of the Pattern-based Mathematics Education Program are compatible with the activities implemented in the school.

During the implementation period of the program, teachers of the children in the experimental group were observed for eight weeks. According to the questions in the Teacher Observation Form, data in observation forms were digitized in frequency and percentage distributions.

Table 6. Findings regarding the observation of the teachers of experimental group children

Items	Teacher -1				Teacher-2			
	Yes		**No**		**Yes**		**No**	
	f	**%**	**f**	**%**	**f**	**%**	**f**	**%**
Preparing the environment	40	100.0	-	-	40	100.0	-	-
Transition	38	95.0	2	5.0	40	100.0	-	-
Developmentally appropriateness	40	100.0	-	-	40	100.0	-	-
Introduction of material	40	100.0	-	-	40	100.0	-	-

Effective use of material	39	97.5	1	2.5	40	100.0	-	-
Application according to the gains in the program	40	100.0	-	-	40	100.0	-	-
Verbal and visual use of language	39	97.5	1	2.5	40	100.0	-	-
Stage appropriateness	36	90.0	4	10.0	38	95.0	2	5.0
Active participation of children	38	95.0	2	5.0	39	97.5	1	2.5
Following the methods and techniques	40	100.0	0	-	40	100.0	-	-
Time management	38	95.0	2	5.0	37	92.5	3	7.5
Drawing attention to the target word	40	100.0	-	-	40	100.0	-	-
Explanation of the target word	40	100.0	-	-	40	100.0	-	-
Active questioning	40	100.0	-	-	40	100.0	-	-
Relating to the daily life experiences	40	100.0	-	-	40	100.0	-	-
Tone of voice and body language	40	100.0	-	-	40	100.0	-	-
Effective communication	40	100.0	-	-	33	82.5	7	18.5
Evaluation	40	100.0	-	-	40	100.0	-	-
Democratic environment	40	100.0	-	-	40	100.0	-	-
Feedback	40	100.0	-	-	40	100.0	-	-

In Table 6, it was observed that Teacher 1 had little difficulty in some activities related to transition to activity, effective use of material, use of verbal and visual language, active participation of children and time management. It was observed that Teacher 2 had more serious problems compared to Teacher 1 in terms of stage appropriateness, effective participation of the children, communication and time management in some activities. It was observed that teachers did not have any difficulties about other items. This situation can be interpreted by the researcher that the information given by the researcher about the education program to the teachers and the feedback given to the teachers after the application was effective before the implementation of the Pattern-based Mathematics Education Program (PMEP) and that it was implemented by the teachers according to the purpose of the program prepared by the researcher.

At the end of the implementation of the Pattern-based Mathematics Education Program, teachers' opinions about the program they applied were evaluated with the "Teacher Interview Form". The teacher was asked the open-ended interview questions which were about the Pattern-based Mathematics Education Program and standardized under the name of Teacher Interview Form. The questions asked and the answers received from the teacher were as follows:

* *Experimental group teachers' opinions about whether Pattern-based Mathematics Program contributed to their personal / professional development*

When the opinions of the experimental group teachers about the education program are examined, Teacher 1 and Teacher 2 have the opinion that the process contributed to their professional development. It can be said that Teacher 2 realized that the patterns could be comprehensively handled with the education program implemented. The reason for this is the answer of Teacher-1 as she said: *"It is obvious that cognitive development studies are important for educators. Basically, working with an educator who prepared this training developed and motivated me"* while Teacher 2 said, *"Yes, I think it is quite effective. I have seen that the pattern I see as superficial can be given in a variety of different ways thanks to the applied activities."* In addition, the absence of any expression of both teachers about their personal development can be interpreted as the education program did not contribute to their personal development.

However, when the opinions of Teacher 1 are examined, it is stated that the education program applied to gain pattern awareness at an early age contributes to the cognitive development of children. As a reason for this situation, it can be shown that Teacher 1's stated her views as: *"Starting patterns and mathematics in the preschool period, which is the first step of education, made significant contributions to the cognitive development of children."*

* *Experimental group teachers' opinions about the difficulties encountered during the implementation of Pattern-based Mathematics Education*

When the teachers' expressions were examined, it was found that they faced difficulty in four major categories as collect the attention of children, difficulties in activities that require strength and skill, difficulties in the gender distribution of the class, and problems in group work. In the category of attention, Teacher-1 said: *"I found that they had difficulty getting their attention from time to time in the integrated pattern activities."* while Teacher 2 said: *"We encountered some difficulties with the short-term attention of the children during the pattern activities that we did as a group."* Based on these statements, it was shown that some activities in the education program were integrated and that small / large group activities were included as a reason for distracting children. In the category of constraint in activities that require strength and skill, Teacher 1's statement as *"Children experienced difficulty in physically demanding activities."* In line with this statement, it can be said that when the pattern skills are desired to be supported by the psychomotor field, it is possible to reflect on the children's psychomotor skills and their ability to design patterns. In the category of non-equality of gender distribution of the class, Teacher 1 expressed her thoughts as: *"The fact that class does not have an equal number of girls-boys distribution caused a decrease in attention and difficulties in activities."* In the category of problems experienced in the group work, as Teacher 2 expressed her views as: *"We have encountered difficulties due to the fact that the children's attention span was short in group work. In the beginning stages, more individual work can be carried out on the table."* When all the expressions are examined, it can be said that children should be supported in the psychomotor area and both teachers had difficulty in class management rather than the program.

As a reason for this, Teacher 1 said: *"Both your being an observer and the fact that the camera was on record affected my authority. It's not enough to know."* while Teacher 2 added: "*I would have lied if I said that the camera didn't affect my teaching, I was worried about making a mistake while you were recording."* These expressions prove that the teachers faced obstacles in class management and introducing the content due to the video recording although they had knowledge about the program and the subject prior to the application.

* *Experimental group teachers' opinions about Pattern-based Mathematics Education*

When the opinions of the teachers about the Pattern-based Mathematics Education Program are evaluated; it is seen that the Pattern-based Mathematics Education Program was implemented successfully, and it contributed greatly to teachers and children. In this sense Teacher 1 said: *"I like the combination of integrated activities. In the context of the pattern, we were able to see all the activities. In addition, I think that the age* *group of my class is in the preparatory phase of primary school and that the use of psycho-motor skills more effectively, especially the repetition of fine motor studies, contributes positively to the development of children."*

In addition, suggestions were made to create awareness about the concept of pattern and to adapt activities to smaller age groups. In this context, Teacher 1 said: *"I think that the age group to start the program was reduced to four or even up to three would increase the effectiveness in terms of education."* while Teacher 2 expressed: "... the applicability of the program on younger age groups could also be possible…"

Apart from these suggestions, there were statements regarding that program would become more comprehensive and effective if teachers and parents were informed more. Teacher 2 explained this as: *"The program might be more comprehensive by organizing in service trainings to preschool and elementary teachers so that children would find more opportunity to deal with patterns."*

Discussion, Conclusion and Suggestions

The aim of this study was to investigate the effect of Pattern-based Mathematics Education Program (PMEP) on the reasoning skills of 61-72-month-old children. For this purpose, it was examined whether there was a significant difference in the Cognitive Ability Test (CogAt-6) pretest scores of the children in the experimental and control groups determined for this purpose. In terms of pretests, both groups were found to be identical.

When the pretest / posttest scores of the children in the experimental group were examined, there was a statistically significant difference between the pretest and posttest scores of all dimensions (Verbal, Numerical and Non-Verbal) ($p < 0.05$). The pretest scores of all children in the experimental group were significantly lower than the posttest scores. This result can be interpreted that the Pattern-based Mathematics Education Program supports the verbal, numerical and nonverbal reasoning skills of children. Similar to our study, Papic, Mulligan and Mitchelmore's (2011) study can be shown. Papic et al. (2011) found a significant difference in favor of the experimental group in their research aiming to examine 45-62-month-old children's development of repeating and visual patterns.

Another result of the study was that there was a statistically significant difference in the Verbal Dimension when the pretest and posttest scores of the children in the control group were examined ($p < 0.05$). The reason for this difference could be the implementation of the activities including gains and indicators for language and cognitive development in the Ministry of National Education (MEB) Preschool Education Program (2013). It can be said that such activities contribute positively to children's verbal reasoning skills. Because, in order to support verbal reasoning skills, it is necessary to support cause-effect relationship, to make sense of rules and principles, to classify concepts, objects or events (Van der Sluis, De Jong and Van der Leij, 2007, pp. 429-430).

Another result in the quantitative part of the study is that there was a statistically significant difference in favor of the experimental group when the numerical dimension posttest scores were examined ($p<0.05$). Patterns include matching, sorting, grouping and comparison skills, and thus children can relate mathematical concepts (Palabıyık and Akkuş-İspir, 2011). Similar to our study, Mulligan, Mitchelmore, Kemp, Marston and Highfield's (2008) study can be shown. Mulligan et al. (2008) investigated the effect of PASMAP (Pattern and Structure Mathematics Awareness Program) on mathematical skills of children aged between four and seven years. At the end of the study, there was a significant difference in the mathematical skills of the in favor of the experimental group. In addition, Lee, Ng, Bull, Pe and Ho (2011) examined whether the development of children's ability to solve algebraic problems that require four procedures in their qualifications is related to their qualifications in pattern, numerical and working memory activities. Research findings indicated that the proficiency of pattern and numerical activities predicted algebraic proficiency. It was stated that the proficiency in both pattern and measurement activities predicted the algebraic capability. It is pointed out that the difficulties in algebraic reasoning may arise from insufficient learning environments (identification and generalization of patterns or inadequate education related to measurement).

Another finding of the study was that the difference between Verbal and Non-verbal Dimension posttest and retention test scores of the children in the experimental group was significant, but the difference between Numerical Dimension posttest and retention scores was not significant ($p > 0.05$). In this case, it can be said that the effect of the education program continued at the end of three weeks in terms of quantitative concepts, Verbal and Non-verbal Dimension. The first four weeks of the Pattern-based Mathematics Education Program included repeating patterns. The repeating patterns included in the education program were given in different presentation forms such as sound, number and shape. The activities in the training program pioneered the development of various simple spatial concepts in the comprehension, reproduction, generalization of repeating patterns and repeating patterns. It has been found that identification and application of repeating unit during the education is important in the perception of children's pattern structure. Mulligan, English, Mitchelmore and Robertson (2010) aimed to assess the effectiveness of the Pattern and Structure Mathematical Awareness Program (PASMAP) for the mathematical development of preschool children. For this purpose, 316 children were given a program for two years. Papic, Mulligan and Mitchelmore (2011), one year after the application studies, found out that the experimental group children 's explaining and continuing skills were better than the control group. The results of Pattern-based Mathematics Education Program are similar in this sense. In the following weeks of the Pattern-based Mathematics Education Program, simple activities related to changing and relationship patterns were included. These activities have allowed the development of multiplicative and functional thinking, pattern and structure awareness (Mulligan and Mitchelmore, 2009). Symbolization of generalization and pattern structure, or simple repetitions, stands out as the points to be taken into consideration when creating a specific learning model on patterning.

When the qualitative findings of the research are examined, it can be said that the Pattern-based Mathematics Education Program was effective in the direction of teacher observations and opinions. It can be stated that the preparation of an education program that handles patterns for young children extensively contributed to the professional development of teachers. There is no doubt that even the smallest awareness of teachers will have positive effects on children. With the slightest change in educational programs, it can be thought that children will benefit from these positive effects on their later learning.

The studies described above are in line with the contribution of Pattern-based Mathematics Education Program to children’s reasoning skills. On the other hand, one of the most important implications of the study is that early age students are not sufficiently encouraged in their preschool experience. Given the chances of mathematical experience in generalizations, it was seen that children were successful in complex concrete patterns before they started elementary school. As the reason for the success of children in complex concrete patterns, the fact that the educator encourages the validation and transfer of patterns in activities can be shown as an important component. When the data obtained in this context are examined, the following recommendations are presented:

- The scope of the Pattern-based Mathematics Education Program can be expanded and adapted to various age groups and the results can be investigated.
- Pattern-based Mathematics Education Program can be applied in preschool education institutions where different socio-cultural level children attend, and comparisons can be made with the obtained data.

• The impact of the Pattern-based Mathematics Education Program can be investigated in terms of different variables such as age, gender, level of education of parents and the duration of attendance to preschool education institutions.

• In order to examine the longitudinal effect of the Pattern-based Mathematics Education Program, children who participated in the implementation can be followed in the first grade and the results can be investigated.

• The Pattern-based Mathematics Education Program for elementary school children can be adapted and their results can be investigated.

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