Creative Mathematics for All?

A Survey of Preservice Teachers' Attitudes

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Abstract

Currently, students who do not show initial promise in mathematics rarely get the opportunity to solve nonroutine problems of novel mathematical content. This paper describes a survey of prospective teachers' opinions about which students should have access to problems requiring creativity and mathematical insight. It also discusses how one could prepare preservice teachers to offer such problems via a methods course and field experience. I propose that such carefully chosen problems should be available to all students, regardless of previous identification as 'gifted'. Historically, non-Asian minorities tend to be underrepresented in gifted programs. Interesting these previously unrecognized students in mathematics could lead to a greater minority presence in the field.

Key Words: Preservice teachers, creative problems, nonroutine problems

Introduction

Nonroutine problems are usually reserved for students identified as gifted or mathematically talented[5]. While it's hard to imagine anyone denying that these students need these types of problems, it also seems likely that these problems could help those not identified as such. Additionally, no method of identifying gifted students is perfect, and those methods are not standard across different states[22]. Widely used current methods of identification lead to underrepresentation by non-Asian minorities[12, 22], although some newer forms are helping to close that gap[14, 17, 19]. Finally, average or below-average students also need problems that stimulate their interests[7, 8, 9, 18, 20]. In particular, when Lourdes College's Pat Schmakel asked urban students what would help them learn better, two of the answers were that problems should be interesting and that they needed challenges at an appropriate level[18].

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Note that giving all students the same type of problems that mathematically talented students receive does not necessarily entail giving them the same problems[21]. As Willingham, a cognitive psychologist at the University of Virginia, says, it is detrimental to give all students the same work, because that work doesn't challenge them equally[20]. However, it is still possible to give the same *kinds* of problems, namely, problems that make students think instead of simply applying an algorithm repeatedly. Ideally, these problems would help improve students' attitudes, thus allowing students to better learn mathematics[6].

Robert Moses is a civil rights activist who created The Algebra Project, a nonprofit group which seeks to ensure that every public school educates all children so they can succeed. Moses argues in his book *radical equations: Civil Rights from Mississippi to the Algebra Project* that traditionally, math education's main purpose was to find and encourage mathematically gifted students who would go on to have mathematically-oriented careers[13]. If one also wishes to focus on encouraging all students to have a basic level of mathematical literacy, the methodology must change. Additionally, literacy can stem from many different areas. For example, Nasir et al. gave traditional problems, like “7/11 = ___%,” and basketball problems, like “You take 11 [free-throw] shots and make seven of them. What's your percentage from the line?” to African American basketball players. They found that if students were given traditional problems first, they did worse on both sets of problems than if they were given basketball problems first[15]. In other words, connections can be made between cultural knowledge and school knowledge if students are introduced to cultural problems first.

Nonroutine problems may engender better attitudes towards mathematics. In her study of African American and Hispanic high school students, Butty found that tenth graders with good attitudes towards mathematics did better mathematically both in tenth and twelfth grades[1]. She also noted that Yair's 1999 study found that traditional mathematical instruction was “especially alienating for Hispanic and African American students”[1]. Nonroutine problems, combined with group-work and inquiry-based learning, may help reduce the racial gap in mathematics achievement.

Many Americans believe that people either possess intrinsic mathematical ability or lack it, and that this is a static quality that cannot be changed[4, 20]. Others think that "math was for some special group of people"[13] or that math is "out of the reach of the 'common' man"[15]. These beliefs are detrimental to students who are not immediately recognized as having promise in mathematics, as they may lead to lowered expectations. Julianne Harm writes that "If equity is our priority, we must have high expectations for all students, regardless of his or her gender, ethnicity, socioeconomic status, or *prior math performance*"[4] (emphasis mine).

Ross and Ericsson, Prietula, and Cokely both found that expertise is acquired, rather than innate[3, 16]. Therefore, it should be possible for average children to achieve at above average levels in mathematics. In 1992, a group of researchers published a study that addressed that idea. In this study, eighth-graders of average achievement were assigned to prealgebra, instead of the usual general math. Teachers had higher expectations for students in these classes, and even though they did not initially differ from those not chosen, at the end of the year, they did the same in some areas and better in others on standardized tests. They also took more advanced math classes in high school and had better grades in those classes[11].

Methods

With this in mind, I surveyed seventeen preservice mathematics teachers about their beliefs concerning what groups should get which kind of problems. I gave extremely brief descriptions of twelve hypothetical students, with every combination of mathematical achievement (above-average, average, or below-average), mathematical enjoyment (enjoys math or doesn't enjoy math), and participation in math class (is attentive and an active participant or doesn't participate). For each student, I asked which combination of problems would be most appropriate: all creative/nonroutine, mostly creative/nonroutine with some algorithmic/routine, half creative/nonroutine and half algorithmic/routine, mostly algorithmic/routine with some creative/nonroutine, or all algorithmic/routine.

Problems cannot always be divided neatly into routine or nonroutine, just like curriculum cannot always be divided into traditional or reform. There may be problems with aspects of both. Additionally, whether a problem is routine depends heavily on the mathematical background of the student solving that problem. A routine eighth-grade algebra problem may be a very nonroutine problem for a third-grader.

Some problems, if given to students who know the relevant mathematics, are undeniably routine or nonroutine, so it is these that we focus on. It is also important to note that not everyone uses the same terminology. Willingham, for instance, simply uses the word "problem" only to mean "cognitive work that poses moderate challenge"[20]. For simplicity, this paper describes problems as either routine and creative or nonroutine and algorithmic.

Routine problems include questions such as, "What are the roots of $f(x) = x^2-5x+4$?" and "Evaluate $2 \times 4^2 - 3 \times (-1)^2$," where the problems can be solved simply by applying the appropriate procedures. Routine problems are often assigned where numbers are changed, but the solution method stays the same. These may produce procedural fluency without conceptual understanding.

Nonroutine problems are more involved. For instance, one might imagine a room with 10 people where everyone shakes everyone else's hand exactly once. How many handshakes are there total? This problem can be solved with a variety of methods. Students are asked to think about the problem and reason through it, rather than simply apply an algorithm. One would not assign the same problem for fifteen different values for the number of people; rather, one would ask students to solve it for a few different values, until they have a general formula, and then move on to the next problem. The reverse problem is also interesting; for example, if there are 17 handshakes, no pair shakes hands twice, and nobody shakes his or her own hand, what is the smallest number of people who could be in the room?

Findings

The results of the survey initially appeared not to be statistically significant. When looking at averages by group, there were several trends that may be of interest for further investigation. Participants were split by gender. There were eight women, six men, one transgendered person, and two who did not give their genders. The transgendered person and the two non-respondents were grouped together into an 'other' category. Age and teaching experience were also examined, but since most of the participants' ages were grouped closely together and most had no experience besides the practicum, the information was not helpful.

One trend is that six of the eight women said that students with above average math achievement should get the most creative problems, followed by those with average achievement, followed by those with below average achievement. Some of those had ties, but most were strict inequalities. The other two women switched the order of the above average and average achievers, while leaving the below average achievers last. More testing would be needed to achieve statistical significance with this finding.

The men and others were fairly scattered on their views. Women's ratings, with one exception, were all between a 2 and a 4 on a 5-point Likert-type scale, with 1 indicating all routine/algorithmic problems and 5 indicating all nonroutine/creative problems. Men ranged from 1.5 to 4.5, and others ranged from 1.25 to 5.

Five participants believed that students who said they did not enjoy math should receive more creative problems than those who said they did. Three more said they should receive equal types of problems, and the remaining nine felt that those who enjoyed math should receive more creative problems. This is close to an even split, as was that of the participation study. Eight participants felt that students who didn't participate should receive more creative problems, four felt they should receive more algorithmic problems, and five felt they should receive equal problems.

Table 1. Enjoyment and Participation Correlations.

Group	Sample Size	Enjoyment Correlation	Enjoyment Significance	Participation Correlation	Participation Significance
Women	8	0.5764	0.1348	0.7447	0.0333
Men	6	-0.7628	0.0776	-0.3440	0.5043
Others	3	0.8237	NA	0.8198	NA

Looking at correlations between scores given for each question gave more data, as shown in Table 1. For both enjoyment and participation, females had positive correlation, indicating that if they give one group creative problems, they are likely to give the other group creative problems, and if they give algorithmic problems to one group, they will likely do the same to the other group. This approached significance for enjoyment and achieved significance for participation. Males, on the other hand, had negative correlations for both, indicating that they tend to believe that different groups should be treated differently. This approached significance for enjoyment and was not significant for participation. The three others were not a large enough group from which to get meaningful correlational results.

Discussion

This data leaves many areas for future research. One of those concerns gender. Are women more likely to believe that below-average achievers need more algorithmic problems than other students, or is that merely an artifact of the small sample size? Repeating the experiment with a larger sample size could help answer such questions.

The sample of 17 was anonymously self-selected from a class of 22. All but one of these 22 students had taken TEAC 451P/851P, Teaching Methods #1, from the same instructor. It is likely that his views helped to shape theirs, so future research should attempt to combat this systematic bias by surveying students from different colleges, and ideally even different states and countries.

The women in this sample had a smaller range than the men-- does that mean that women tend to believe that curricula should be more balanced between creative and algorithmic problems? Using a seven-point Likert-type scale instead of a five-point scale might allow for more information on that to be gathered; as it was, very few chose either extreme.

Another area for future study is whether these beliefs change over time. All of these students had just completed a practicum in secondary mathematics education. A few had completed other practica in the past or had other teaching experiences, but none had spent extended time in charge of a secondary mathematics classroom. A follow-up study might ask these students' opinions after student teaching and a year or two of regular classroom teaching. Alternatively, one might sample a different group of more experienced teachers, such as those who have taught for at least ten years.

Lastly, it is unknown how much these beliefs actually impact the teachers' actions. For instance, a teacher might believe that students should get creative problems, but feel unable to give them such problems due to time constraints or being unable to find those problems. A study that compares teachers' beliefs with their practices could be quite revealing. Other studies have shown that creative problems tend to be reserved for the above average achievers[2], so if that conflicts with teachers' beliefs, it is important to find out why and to address that conflict.

Conclusions

The first step in preparing preservice teachers to offer creative problems is to convince them that such problems are important. Without that belief, teachers are unlikely to continue offering nonroutine problems[10]. Therefore, it is advisable to share some of the research literature with them. For instance, they might read Lubienski's work, where she plainly states, "Instead of having students complete meaningless exercises and memorize what the teacher tells them, why not have students learn key mathematical ideas while solving interesting problems?"[8] Lubienski, an associate professor at the University of

Illinois, adds that such problems seem appropriate for all students. Moses echoes that, saying that students can still practice basic concepts while working on more advanced problems[13].

Teachers may fear that nonroutine problems cannot teach the same material that routine problems do. At this point, however, it is not the complete replacement of routine problems that is advocated. While that might be an eventual goal, it is unclear whether that would benefit students, and more research is needed.

Once preservice teachers believe nonroutine problems are important, they need access to some examples. These examples should not be considered their entire pool from which to draw, but merely as possibilities designed to encourage teachers to develop their own. Therefore, I propose focusing on nonroutine problems in courses on teaching methods, including giving advice about generating or finding such problems. Ideally, preservice teachers will then be able to share some of these nonroutine problems with their classes, and they can observe for themselves how students react.

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