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# Definitions and Forms of Certain 2D Geometrical Shapes as Provided by Fourth Year Pre-service Primary School Teachers

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ABSTRACT

Geometry is one of the sub-fields of learning in mathematics. The mathematics curriculum of primary schools attaches great importance to the identification and naming of geometric shapes and objects, the categorization of geometric concepts, and the awareness and depiction of the characteristics of concepts. Applying a descriptive design, the present study seeks to understand how fourth year pre-service primary school teachers define geometric shapes, and how they draw such shapes on the basis of their conceptualization. A total of 88 fourth year pre-service teachers enrolled in the Primary School Teacher Training program of the Faculty of Education at a state university in eastern Turkey were included in the study. Pre-service teachers were issued a form asking for the definition and drawing of 12 concepts in geometry (angle, polygon, triangle, quadrilateral, trapezoid, parallelogram, rhombus, rectangle, square, deltoid, circle, and closed circular region). The results suggest that pre-service teachers experience difficulties in providing correct definitions and drawings. Moreover, an in-depth review of the definitions and drawings provided by the pre-service teachers with respect to geometric concepts reveals that they struggle more with providing correct definitions than with making drawings. Therefore, it can be recommended that activities be developed to improve pre-service teachers' content knowledge throughout the course of their university education.



Keywords:
geometrical shapes, definitions, drawings, pre-service primary school teachers

## Introduction

Geometry, a mathematical field which focuses on shapes and objects, is one of the principal fields of mathematics and one we commonly encounter in our daily life. For this reason, it is seen as being a functional tool in terms of providing practical solutions to problems. Educational attainments in the field of geometry are expected to be achieved through the course of education, not only from formal school but also from the mathematics curricula offered in other education programs (MEB, 2015). However, with that said, geometry is among the topics of which students in any stage of education report having difficulties (Halat, 2008; Yenilmez & Korkmaz, 2013). In particular, students tend to have difficulties with the definitions of concepts in geometry and fail to develop a coherent understanding of the most fundamental concepts of geometry and the relationships between such concepts (Yenilmez & Yaşa, 2008). The lack of a command of the concepts therefore is one of the chief shortcomings seen in students.

One of the most critical components of mathematics knowledge is an awareness of the definitions of mathematics concepts. Definitions are particularly integral parts of advanced mathematics education, as they enable the delimitation of the concepts taught in the classes, help identify applicable and non-applicable examples, facilitate the development of proofs (Ashgari, 2004), and allow students to employ terms in a useful

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and consistent way when reading mathematics. Moreover, it is crucial that the students have an awareness of the meaningful, correct, and applicable definitions of the concepts pertaining to the terminology of mathematics in order for them to be capable of understanding texts entailing mathematical contents (Adams, 2003). That is why language plays such a vital role in teaching and learning mathematics (Austin & Howson, 1979; Gray, Pinto, Pitta & Tall, 1999). Rendering the definition and description of abstract concepts comprehensible for the students requires the use of a correct language of mathematics, for mathematics in general and geometry in particular entail numerous concepts and terms (Aiken, 1972).

Geometry is one of the sub-fields of learning in mathematics. The mathematics curriculum of primary schools attaches great importance to identifying and naming geometric shapes and objects, categorizing geometric concepts, having an awareness of the characteristics of concepts and being able to draw them. A justification for the importance thus attached lies in the idea that students will be able to more easily establish links between the objects around them and the abstract concepts they learn. Moreover, with this curriculum being focused on these areas, students' understanding of and proficiency in geometry will be enhanced (Çetin & Dane, 2004). Primary and secondary school mathematics teachers play a major role in developing these skills envisaged by the curriculum (Cilavdaroğlu, 2012). However, even with this curriculum in place, the students still fail to achieve a complete understanding of the most fundamental concepts of geometry and the relationships between such concepts. As a result, there is a substantial gap between the level of geometric thought and the level of geometry proficiency expected of the students (van Hiele, 1999). It is therefore necessary to instill field knowledge to help teachers close that gap.

Teachers are expected to have a strong grasp of the concepts and contents employed in mathematics teaching, and to communicate accurate and applicable statements to the students when applying these concepts and contents. This is important insofar as the teachers' command of the field contributes positively to the students' success in mathematics (Baki & Çelik, 2005; Hill, Rowan & Ball, 2005; Jones, 2000). Shulman (1987) noted that the various types of field knowledge a teacher is expected to have involve references to basic concepts and principles regarding that field, the curriculum, and the relationships between these. Furthermore, as the teachers' behavior in the classroom and their practices in the activities therein are defined with reference to field knowledge (Ball & Bass, 2000), field knowledge plays a major role in the education process taking place in the classroom (Baki & Çelik, 2005; Ball, Lubienski & Mewborn, 2001; Çakan, 2004; Dursun & Dede, 2004; Seferoğlu, 2001). In this context, the knowledge to be provided and the means of providing that knowledge are important, for the effectiveness of the teacher is among the major factors affecting the ability of students to learn (Romberg & Carpenter, 1986). The teachers' grasp of definitions can play a direct role in the planning of the education process, the management of the learning experiences, the mathematical language employed in the classroom, and the use of the applicable assessment and evaluation methods and techniques (Baştürk, 2009). Teachers' shortcomings in terms of their grasp of definitions and their misperceptions may lead to deficiencies and/or inaccuracies in students' learning as well as to the development of misconceptions (Cilavdaroğlu, 2012). These deficiencies in teachers' grasp of definitions also have direct implications on numerous pedagogical skills, such as the planning of the teaching process and the management of the learning-teaching experiences (Ball, 2000; Baştürk, 2009; Shulman, 1987). That is why the teachers' grasp of definitions and knowledge of shapes should be identified and their errors and misconceptions in this field determined, followed by measures to eliminate such problems.

It is critical that pre-service teachers, as prospective teachers of the future, be trained to have an awareness of the concept of definition (Cilavdaroğlu, 2012). This requires that, if necessary, their existing understanding of definitions and forms be reestablished. A review of the literature has, however, revealed that the relevant studies have largely focused more on the definition and form knowledge of the students as opposed to that of the teachers (Akkaya & Durmuş, 2010; Dağlı & Peker, 2012; Güngörmüş, 2002; Kesicioğlu, Alisinanoğlu & Tuncer, 2011; Kiriş, 2008; Öksüz, 2010; Tunç & Durmuş, 2012; Ubuz, 1999; Van der Sandt & Nieuwoudt, 2005; Vinner & Dreyfus, 1989; Yaman & Şahin, 2014; Yenilmez & Yaşa, 2008). There is a dearth of studies that have centered on the impact of the knowledge and understanding of geometric concepts by pre-service teachers, in contrast to the plethora of studies on this same subject matter as applied to students at different years of primary and secondary schools (Cansız Aktaş & Aktaş, 2012; Cilavdaroğlu, 2012; Dane & Başkurt, 2012; Gürbüz & Gülburnu, 2013; Kesicioğlu, Alisinanoğlu & Tuncer, 2011; Öksüz, 2010; Yenilmez & Yaşa, 2008). While there has been research performed with pre-service teachers, including the study conducted

by Cilavdaroğlu (2012) with first-year pre-service elementary mathematics teachers, as well as the one by Fujita (2012), Fujita and Jones (2007, 2006) with primary trainee teachers, studies with pre-service primary school teachers are nonetheless relatively few and far between. The limited number of studies focused on pre-service teachings is particularly concerning though, considering that primary school teachers are expected to have accurate and comprehensive images of concepts for effective teaching (Duatepe Paksu, Musan, İymen & Pakmak, 2012).

In order to foster meaningful learning among the students, the teacher needs to accurately present the definitions of the concepts and employ visualization and materialization techniques with drawings, graphs, and examples from daily life. For teachers to be able to teach such concepts in a way students can understand, they should have sufficient proficiency in the field. That is why an investigation of the definition and drawing proficiencies of pre-service teachers, who will assume teaching positions in the future, regarding basic concepts of geometry, is deemed crucial (Cilavdaroğlu, 2012), particularly considering that pre-service teachers were found to have low levels of content knowledge regarding geometry (Duatepe Paksu, İymen & Pakmak, 2013), and a readiness level of just 56% in geometry (Duatepe Paksu, 2013). The concepts pre-service teachers hold with respect to geometric shapes will certainly affect the conceptualizations future students will have. That is why the present study attempts to discover how fourth year pre-service teachers define the concepts pertaining to geometric shapes and how they draw such shapes, and reviews the level of accuracy of their definitions and drawings. In this vein, the research problem the study investigates is expressed as follows:

How do fourth year pre-service primary school teachers define the concepts pertaining to geometric shapes and how do they draw them? How accurate are their definitions and drawings concerning geometric shapes?

## Method

The present study was designed as descriptive. Descriptive research emphasizes the similarities and differences regarding the characteristics of the individuals or groups under review, and seeks to present them as conclusions (Gall, Borg & Gall, 1996). This study will, in this context, attempt to present the pre-service teachers' definitions and drawings regarding geometric shapes. A total of 88 fourth-year pre-service teachers, enrolled in the Primary School Teacher Training program of a Faculty of Education at a state university in eastern Turkey during the spring semester of the 2014-2015 academic year, took part in the study. Among the pre-service teachers, 63 (72%) were female and 25 (28%) male.

### Data Collection Tool

For the study, pre-service primary school teachers were issued a form asking them to provide a definition and drawing of 12 concepts in geometry (angle, polygon, triangle, quadrilateral, trapezoid, parallelogram, rhombus, rectangle, square, deltoid, circle, closed circular region). On the form, each concept was separately written, with a blank space beside it for the students to write the definitions and draw the shapes related to the concept. The form employed in the study was developed by adding the concepts of quadrilateral and the closed circular region to the basic concepts of geometry form developed by Cilavdaroğlu (2012). The participants were then asked to provide a definition for each geometric concept presented in the form, and to drawn the shape representing that concept. The pre-service teachers were presented the form in an exam-like setting, so as to prevent undue influence from other participants, and were given 90 minutes to fill out the form.

### Data Analysis

The analysis of the responses provided on the form used in the study to gather data was based on a scale of 4 categories, developed by adding "blank" to the 3 categories established originally by Cilavdaroğlu (2012) –"correct, partly correct, and incorrect"–, on the basis of the review of the definitions provided in the literature with respect to geometric concepts. For this, the different definitions of the concepts in the literature were initially examined. Moreover, the common points of these definitions were identified. Where the definitions and drawings provided by pre-service teachers were very close to the definitions and drawings provided in the literature, and where they were complete and accurate, they were labeled "correct". The definitions and drawings with minor errors or deficiencies were labeled "partly correct", meaning, in other

words, that such definitions and drawings provided only a partial description of the concept. Inaccurate definitions and drawings, in turn, were assigned to the incorrect category. Lastly, in cases where a participant skipped a question asking for the definition or drawing of a certain concept, a "blank" label was applied. The responses were analyzed separately by two researchers, after which the analyses were compared. Items on where there was consensus between the two analyses were accepted directly. On the basis of this format of analysis, the reliability factor was found to be high, having a value of 0.83, as per Miles and Huberman's (1994; p. 64) formula (Reliability Factor=Number of Consensus Items/(Number of Consensus Items+Number of Divergence Items). Finally, the analyses were compared by both researchers together in order to come up with a joint assessment regarding the points of divergence.

## Results

The present study attempts to discover how fourth year pre-service teachers define the concepts pertaining to geometric shapes and draw such shapes, and to review the level of accuracy of their definitions and drawings. This section of the article will present the collected findings.

Table 1 presents a summary of the definitions and drawings pre-service primary school teachers provided for the concepts of angle, polygon, triangle, quadrilateral, trapezoid, parallelogram, rhombus, rectangle, square, deltoid, circle, and closed circular region, under the blank, incorrect, partly correct, and correct categories, along with the percentage for each category.

**Table 1.** The definitions and drawings provided by pre-service teachers with reference to geometric shapes.

| Concept | Definition | | | | Drawing | | | |
|---|---|---|---|---|---|---|---|---|
| | Blank (%) | Incorrect (%) | Partly Correct (%) | Correct (%) | Blank (%) | Incorrect (%) | Partly Correct (%) | Correct (%) |
| Angle | 4 | 18 | 71 | 7 | - | - | 99 | 1 |
| Polygon | 2 | 1 | 73 | 21 | 10 | - | 85 | 5 |
| Triangle | 1 | 10 | 41 | 48 | - | - | - | 100 |
| Quadrilateral | 2 | 6 | 53 | 39 | - | - | - | 100 |
| Trapezoid | 5 | 23 | 34 | 38 | 1 | 6 | 75 | 18 |
| Parallelogram | 2 | 7 | 45 | 46 | 1 | - | 94 | 5 |
| Rhombus | 5 | 16 | 59 | 20 | - | 18 | 34 | 48 |
| Rectangle | 2 | 13 | 46 | 39 | - | 1 | 47 | 52 |
| Square | 4 | 6 | 55 | 35 | 1 | 1 | 44 | 54 |
| Deltoid | 24 | 12 | 34 | 30 | 6 | 29 | 25 | 40 |
| Circle | 13 | 19 | 31 | 37 | 1 | 3 | - | 96 |
| Closed Circular Region | 17 | 7 | 68 | 8 | 4 | 16 | - | 80 |

A glance at Table 1 reveals that the definitions and drawings provided for each geometric shape exhibit variations in terms of their distribution to the categories of blank, incorrect, partly correct, and correct. The geometric shapes with the highest rate of being skipped, in terms of the definition, were deltoid (24%), closed circular region (17%), and circle (13%), while the ones with the lowest rate of blank responses were triangle (1%), polygon (2%), quadrilateral (2%), parallelogram (2%), and rectangle (2%). The concepts which received the highest rate of incorrect definitions were trapezoid (23%), circle (19%), angle (18%), and rhombus (16%), while the lowest rate of incorrect definitions had occurred with polygons (1%), quadrilaterals (6%), and squares (6%). Generally, the highest percentage of responses were placed under the partly correct category. The geometric shape that was most often defined correctly was the triangle (48%), while the one that received the lowest rate of correct definitions was angle (7%). The shape for which the highest number of participants failed to draw the concept was the polygon (10%), while the shape which received the highest number of incorrect drawings was deltoid (29%). Furthermore, 99% of the participants provided partly correct drawings

of angle, while triangle and quadrilateral generated a 100% accurate drawing rate. The shape for which the lowest rate of accurate drawings was observed was angle (1%).

Eighteen percent of the definitions pre-service primary school teachers provided with respect to the concept of angle were incorrect, while 7% of them provided correct definitions, 4% skipped the question, and 71% provided partly correct definitions. To cite an example of an incorrect definition, one of the participants defined angle as "the region between 2 straight lines" (Ö47). The drawing by the same participant (Ö47') was also incorrect. On the other hand, the definition provided by the Ö6-designated participant was reflected in their accurate drawing as well. The correct definition of angle as provided by Ö6 was "the shape produced as two beams joined by a shared starting point", accompanied by the correct drawing, as shown in Figure 1.

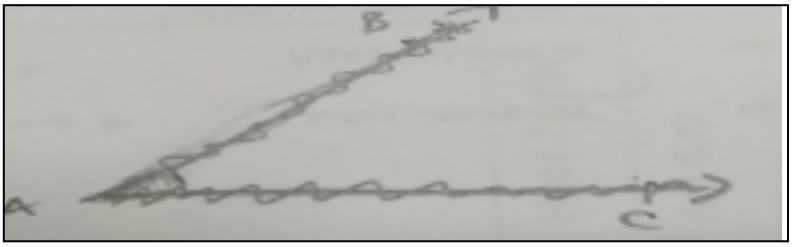


**Figure 1.** Ö6's drawing regarding the concept of angle.

Figure 1 shows that Ö6 provided a correct drawing by emphasizing the fact that angle refers to the geometric shape formed by the joining of 2 beams with a shared starting point. The results showed that 99% of the participants drew the concept of angle correctly, while 1% provided an incorrect drawing.

A total of 73% of the definitions pre-service teachers provided for the concept of polygon were partly correct, while blank, incorrect, and correct responses amounted to 2%, 1%, and 21%, respectively. To cite an example of a partly correct definition for polygon, one of the partipants wrote, "A shape with at least 3 sides" (Ö73). Said participant's drawing reflected this definition but included only triangular, quadrilateral and pentagonal concepts. A total of 5% of the drawings provided by pre-service teachers with respect to the concept of polygon were correct, while partly correct drawings were provided by 85%, and 10% skipped the question. An example of a correct drawing (Ö63) is provided in Figure 2.


**Figure 2.** Ö63's drawing regarding the concept of polygon.

Figure 2 reveals that Ö63 emphasized the possibility of having n sides in a polygon in the drawing pertaining to that concept, and hence provided a correct drawing. A total of 48% of the definitions pre-service teachers provided for the concept of triangle were correct, while blank, incorrect, and partly correct responses amounted to 1%, 10%, and 41%, respectively. As an example of a correct definition provided, one participant wrote, "Closed shape with 3 sides and angles, where the sum of interior angles is 180 ”°(Ö24). Moreover, 100% of the drawings provided by pre-service teachers with respect to the concept of triangle were correct. The drawing provided by Ö53 is presented below as an example of an accurate drawing.

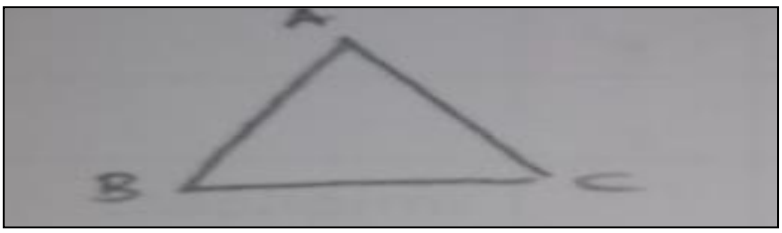


**Figure 3.** Ö53's drawing regarding the concept of triangle.

Figure 3 demonstrates that Ö53 emphasized the triangle's characteristic of being a closed shape with three sides in the drawing pertaining to that concept, and hence provided a correct drawing. A total of 53% of the definitions pre-service teachers provided for the concept of quadrilateral were partly correct, while blank, incorrect, and correct responses amounted to 2%, 6%, and 39%, respectively. As an example of one of the partly correct definitions provided, participant Ö62 wrote: "A figure with 4 sides, the opposite sides of which run parallel to each other". Although the definition provided by Ö62 was accepted as being only partly correct, the drawing was acknowledged to be correct. Among the drawings provided by pre-service teachers for the concept of quadrilateral, 100% were correct. As an example of a correct drawing, the quadrilateral provided by participant Ö2 is shown in Figure 4 below.

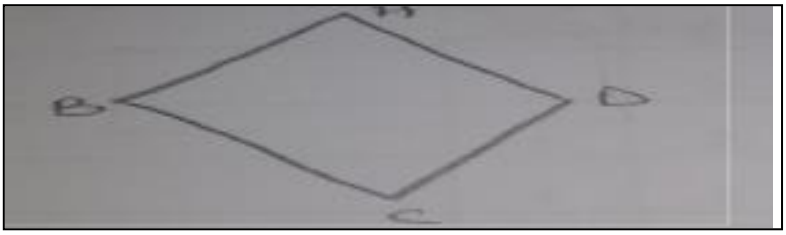

**Figure 4.** Ö2's drawing regarding the concept of quadrilateral.

Figure 4 reveals that Ö2 emphasized the quadrilateral's characteristic of being a closed shape with four sides in the drawing pertaining to that concept, and hence provided a correct drawing. A total of 5% of the pre-service teachers skipped the question asking for the definition of the concept of trapezoid, while incorrect, partly correct, and correct responses amounted to 23%, 34%, and 38%, respectively. One of the partly correct definitions provided was "A shape with 2 sides running in parallel", provided by Ö2. Although the definition provided by Ö2 was partly correct, the drawing was correct.

Among the pre-service teachers, 1% left blank the question asking for a drawing of the concept of trapezoid, while incorrect, partly correct, and correct drawings were provided by 6%, 75%, and 18% of the pre-service teachers, respectively. An example of an incorrect drawing, provided by Ö28, is presented in Figure 5. The definition of the trapezoid provided by Ö5 was also incorrect.

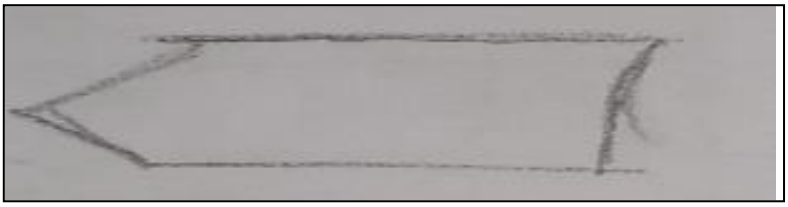

**Figure 5.** Ö28's drawing regarding the concept of trapezoid.

Figure 5 reveals that Ö28 provided an incorrect drawing by emphasizing an irregular polygon in response to the question asking for a drawing of the concept of trapezoid.

A total of 2% of the pre-service teachers skipped the question asking for the definition of the concept of parallelogram, while incorrect, partly correct, and correct responses amounted to 7%, 45%, and 46%, respectively. One of the incorrect definitions was provided by Ö70, who noted that it was "A shape produced by the intersection of parallel sides". Although the definition provided by Ö70 was incorrect, the drawing was partly correct.

A total of 1% of the pre-service teachers left blank the question asking for a drawing of the concept of parallelogram, while partly correct and correct drawings were provided by 94% and 5% of the pre-service teachers, respectively. Ö33's drawing, as an example of a partly correct response, is provided in Figure 6. The definition of the parallelogram provided by Ö 33 was incorrect.

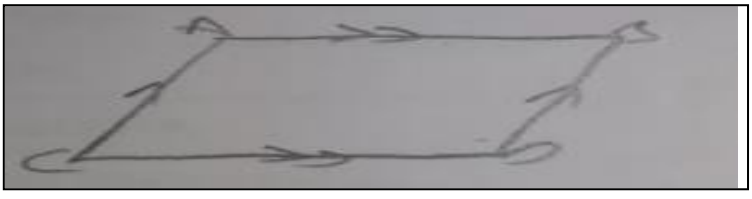

**Figure 6.** Ö33's drawing regarding the concept of parallelogram.

When drawing the shape describing the parallelogram concept, Ö33 emphasized only the parallel nature of the opposite sides of the parallelogram, providing, hence, a partly correct drawing (see Figure 6). Yet, such a description is not a complete reflection of the concept of parallelogram. Other geometric shapes such as rectangles and squares also have their opposite sides running in parallel to each other.

A total of 5% of the pre-service teachers skipped the question asking for the definition of the concept of rhombus, while incorrect, partly correct, and correct responses amounted to 16%, 59%, and 20%, respectively. One of the incorrect definitions provided by the participants was the one offered by Ö28, which defined rhombus as "A shape, the opposing 2 sides of which match". Both the definition and the drawing provided by Ö28 were incorrect.

A total of 18% of the pre-service teachers provided an incorrect drawing of the concept of rhombus, while partly correct and correct drawings were provided by 34% and 48% of the pre-service teachers, respectively. Ö82's drawing provided below serves as an example of an incorrect depiction (see Figure 7). The definition of the rhombus provided by Ö82 was also incorrect.

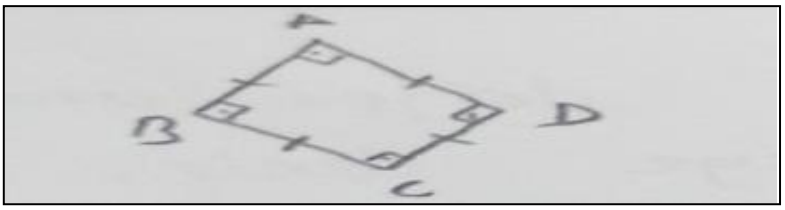


**Figure 7.** Ö82's drawing regarding the concept of rhombus.

Figure 7 reveals that Ö82 provided an incorrect drawing by emphasizing the form of a square in response to the question asking for a drawing of the concept of rhombus. A total of 2% of the pre-service teachers skipped the question asking for the definition of the concept of rectangle, while incorrect, partly correct, and correct responses amounted to 13%, 46%, and 39%, respectively. One of the partly correct definitions provided was "The shape with matching opposite sides and internal angles of 90 ″°(Ö15). This definition was categorized as only partly correct, as it also matches the concept of square. Although the definition provided by Ö15 was partly correct, the drawing was correct.

A total of 1% of the pre-service teachers provided an incorrect response to the question asking for a drawing of the concept of rectangle, while partly correct and correct drawings were provided by 47% and 52% of the pre-service teachers, respectively. Ö5's drawing as an example of a partly correct response is provided in Figure 8. The definition of the rectangle provided by Ö5 was also partly correct.

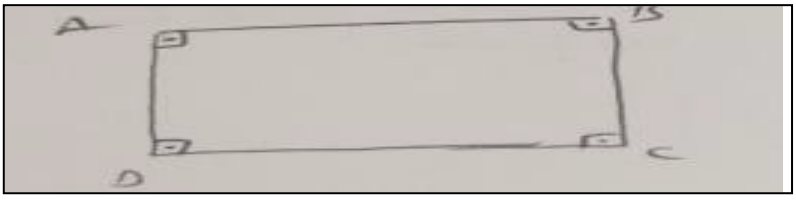


**Figure 8.** Ö5's drawing regarding the concept of rectangle.

Figure 8 reveals that Ö5 was content with merely providing a prototype of the rectangle, emphasizing that its internal angles are 90° when providing a drawing of the concept of rectangle. The drawing was categorized as partly correct, for it did not specifically refer to the identifying characteristics of the concept of rectangle, but instead corresponded to the definitions of many other geometric concepts as well.

A total of 4% of the pre-service teachers skipped the question asking for the definition of the concept of square, while incorrect, partly correct, and correct responses amounted to 6%, 55%, and 35%, respectively. One of the correct definitions was provided by Ö80, who defined square as a "quadrilateral with equal sides and angles of 90°". The drawing provided by Ö80 was also correct.

A total of 1% of the pre-service teachers left blank the question asking for a drawing of the concept of square, while incorrect, partly correct, and correct drawings were provided by 1%, 44%, and 54% of the pre-

service teachers, respectively. The partly correct drawing provided by Ö18 is provided below (see Figure 9). Said participant's definition of the square was correct, however.

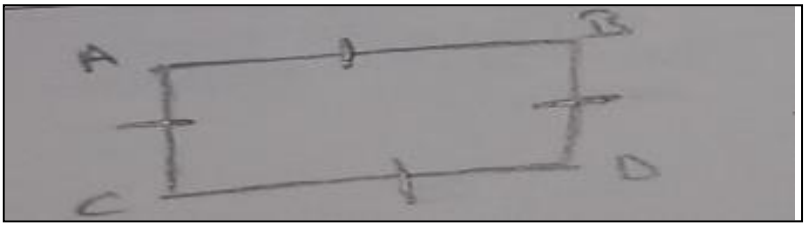


**Figure 9.** Ö18's drawing regarding the concept of square.

Figure 9 reveals that Ö18 merely provided the prototype of the square and emphasized that it had matching sides, hence offering only a partly correct drawing. A total of 34% of the pre-service teachers skipped the question asking for the definition of the concept of deltoid, while incorrect, partly correct, and correct responses amounted to 12%, 34%, and 30%, respectively. One of the partly correct definitions was provided by Ö2, who described it as a "shape composed of the combination of two isosceles triangles". The drawing provided by Ö2 was also partly correct.

A total of 6% of the pre-service teachers skipped the question asking for a drawing of the concept of deltoid, while incorrect, partly correct, and correct responses amounted to 29%, 25%, and 40%, respectively. Ö58's drawing as an example of a correct one is provided in Figure 10. The definition provided by Ö58 was also correct.

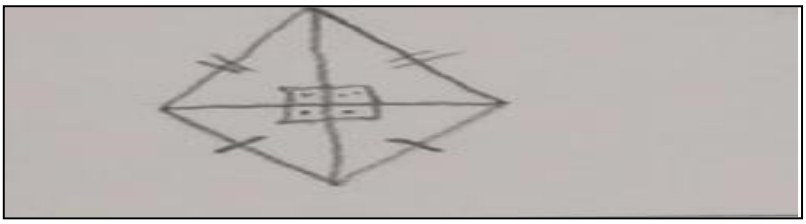

**Figure 10.** Ö58's drawing regarding the concept of deltoid.

The shape drawn by Ö58 corresponding to the concept of deltoid was correct insofar as it emphasized that the deltoid is formed by the combination of two isosceles triangles on their bottom sides, and that their diagonals intersect at a right angle (see Figure 10).

A total of 19% of the pre-service teachers who took part in the study provided incorrect definitions of the circle concept, while 31% provided partly correct definitions, 37% provided correct definitions, and 13% provided no definition at all. One of the incorrect definitions was provided by Ö21, who defined it as "A shape formed by the combination of an indefinite number of points". Although the definition provided by Ö21 was incorrect, the drawing was correct.

A total of 3% of the pre-service teachers provided incorrect drawings, while 96% provided correct drawings, and 1% skipped the question asking for a drawing of a circle. Ö55 was among the pre-service teachers who provided an incorrect drawing of the circle. The shape drawn by the said participant is provided below (see Figure 11). The definition provided by Ö55 was also incorrect.

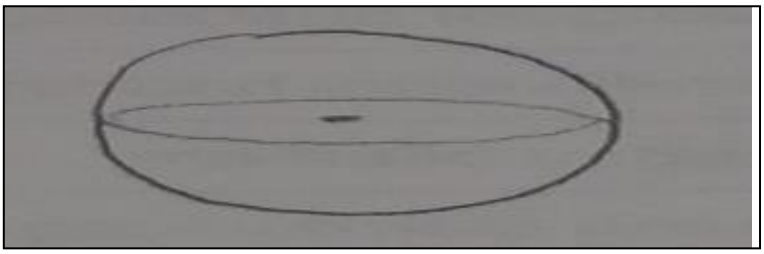

**Figure 11.** Ö55's drawing regarding the concept of circle.

Figure 11 shows that Ö55 incorrectly represented the circle as a 3D object that effectively amounts to a sphere. Yet the participants were asked to draw a circle. The last concept included in the study was the closed

circular region. A total of 7% of the pre-service teachers provided an incorrect definition in response to the question asking for the definition of the concept of closed circular region, while partly correct and correct responses amounted to 68% and 8%, respectively. Furthermore, 17% of the participants provided no definition at all. One of the partly incorrect definitions was provided by Ö52, who defined closed circular region as "A filled shape". Although the definition provided by Ö52 was partly correct, the drawing was correct.

A total of 16% of the pre-service teachers provided incorrect drawings, while 80% provided correct drawings, and 4% skipped the question asking for a drawing of the closed circular region. Ö74's drawing as an example of an incorrect one is provided in Figure 12. In addition, the drawing provided by Ö74 is also incorrect.

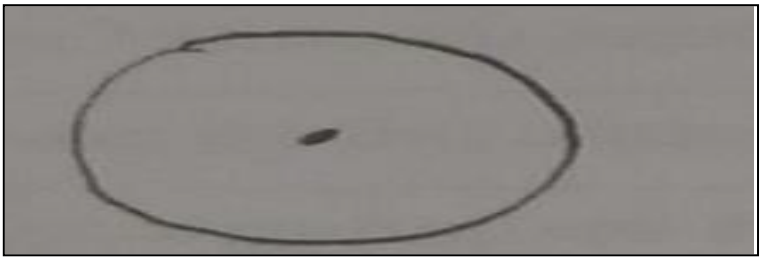

**Figure 12.** Ö74's drawing regarding the concept of closed circular region.

Ö74's drawing above fails to emphasize that the closed circular region is actually a filled shape. The shape drawn by Ö74 is that of a circle, rather than that of a closed circular region.

## Discussion and Conclusions

Geometry is a field with concrete links to our daily life and constitutes a significant part of the mathematics curricula at all levels. The basics of geometry education are laid down by primary school teachers, and that is why it is crucial for them to have sufficient expertise. In this vein, the present study has sought to understand how fourth year pre-service primary school teachers define geometric shapes, and how they draw such shapes on the basis of their conceptualization.

The results suggest that pre-service teachers have difficulties in providing correct definitions and drawings, and more often than not provide only partly correct definitions and drawings. The lack of a conceptual grasp of the geometric shapes on the part of the pre-service teachers may be among the obstacles preventing them from providing correct definitions and drawings (Linchevsky, Vinner & Karsenty, 1992). An overall look at the definitions provided by pre-service teachers with respect to geometric concepts reveal that the definitions provided are rather superficial. In previous studies conducted by Fujita and Jones (2006, 2007), it was reported that the pre-service teachers 'knowledge about basic geometric shapes was limited, and that these teachers had difficulty defining and classifying quadrilaterals in particular. Moreover, Cunningham and Roberts (2010) stated in their study that even though the definitions of concepts were provided, most of the pre-service teachers failed at interpreting these definitions and at putting them into practice in new cases (including nonstandard examples). A study conducted by Fujita (2012) with 19 trainee teachers found that more than half of the participants assessed quadrilaterals in terms of prototype examples/shapes, although they did know the accurate definition. The pre-service teachers' shortcomings in terms of effective use of mathematics lingo may be one of the reasons to account for their provision of such definitions. Yet, language plays a crucial role in the learning and teaching of mathematics. The correct use of mathematics language is one of the most effective means for teaching knowledge of mathematical concepts, and it plays a major role in ensuring that the students learn the concepts better (Gray, Pinto, Pitta & Tall, 1999).

An in-depth review of the definitions and drawings provided by pre-service teachers with respect to geometric concepts reveals that they have more issues with providing correct definitions than with producing drawings. Other studies in the literature also noted problems related to definitions (Çetin & Dane, 2004; Cilavdaroğlu, 2012; Kaplan & Hızarcı, 2005; Tunç & Durmuş, 2012). The higher level of success achieved with drawings may be explained by the pre-service teachers' use of standard geometric shapes when drawing such shapes in their own geometry classes, the results of which lead to them embracing such shapes in their subconscious mind (Güven, 2002). Yet another explanation may lie in the pre-service teachers' shortcomings

regarding the conceptual learning of the concepts presented to them and their deficiencies regarding the use of the mathematics language and terminology (Cilavdaroğlu, 2012).

Another striking finding is the significant level of failure pre-service teachers have in providing correct definitions and drawings of the concept of angle, as was seen in Cilavdaroğlu's (2012) study. Some pre-service teachers were observed to confuse the angle itself with the region covered by the angle when providing the definition, while others used the concepts of angle and its protraction interchangeably, disregarding the fact that angle is a geometric shape formed by two beams sharing the same starting point. When providing the definition and drawing of the concept of angle, pre-service teachers were observed to often emphasize the concepts of region-area-space.

Moreover, pre-service teachers were observed to have the highest rates of failure with the drawings of the concepts of polygon, parallelogram, and trapezoid. The drawings of the concept of polygon usually disregarded the possibility of having n sides in a polygon, and instead opted for a polygon with five or six sides, like 7th grade students tend to do, as evidenced in the study by Ergün (2010), which found that 7th grade students had a perception of the concept of polygon similar to that seen in the pre-service teachers in the present study. A study conducted by Erşen and Karakuş (2013) to assess the concept images of pre-service primary school teachers for quadrilaterals found that these pre-service teachers made drawings inaccurately because they lacked notation representation in the drawings of quadrilaterals; they did not know the shape's features and could not classify the relations between quadrilaterals; and they had misconceptions in their individual definition for quadrilaterals. The drawings for the parallelogram, in turn, exhibited only the prototype of the shape and did not emphasize the identifying characteristics of it. Ubuz (1999) reached a similar result, reporting that 10th and 11th grade students considered a parallelogram to be a quadrilateral with opposite sides running parallel to each other. It is likely then that these students would begin university with an inaccurate understanding of the drawings of these concepts. The drawings regarding the concept of trapezoid, in turn, provide a classification of the irregular shapes under this definition. The pre-service teachers may have based their drawings on the connotations that the word "yamuk" –the Turkish term for trapezoid –has, and this, therefore, may signify a misconception on the part of the pre-service teachers; for the studies show that this is one of the recurring misconceptions observed with the concept of trapezoid (Doğan, Özkan, Çakır, Karlı, Baysal & Gün, 2012). Returning to the study by Erşen and Karakuş (2013), it was found that pre-service primary school teachers had misconceptions about trapezoids. Although pre-service teachers had difficulties in providing correct definitions of the concept of closed circular region, they did not have such issues when providing drawings thereof. Cilavdaroğlu (2012) also notes a similar finding. This is perhaps due to the disregard of the true definition of closed circular region, which is the filled shape formed by merging the points located at a specific distance from a fixed point, in favor of the definition of merely a filled shape. In addition, the drawings had emphasized only the filled nature of the closed circular region, a finding likely able to be attributed to the experiences the participants had in their geometry classes. Indeed, numerous shortcomings regarding the pre-service teachers' content knowledge may account for these results.

Content knowledge is noted to be among the most important elements affecting the in-class practices of the teachers (Ball & Bass, 2000; Ball, Lubienski & Mewborn, 2001). Therefore, it can be recommended that activities be implemented throughout the course of university education to improve pre-service teachers' content knowledge. The lack of field knowledge results in pre-service teachers having difficulties in defining the concepts, and thereby failing to achieve conceptual learning. To prevent this, courses tasked with training pre-service teachers to attain conceptual learning can be developed. During these courses, drawing exercises of the concepts can be included. It is suggested that future studies on this topic conduct more detailed investigations of pre-service teachers' content knowledge levels by performing interviews focusing on specific concepts.

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