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Diagnosing Learning Difficulties Related to the Equation and Inequality that Contain Terms with Absolute Value

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Abstract

The aim of this study is to diagnose the learning difficulties about the equation and inequality that contain terms with absolute value and to make suggestions for the teachers in this respect. The sample of the research is composed of 170 ninth grade students enrolled in four different high schools. Data of the research is composed of a knowledge test that contains 10 open-ended questions and interviews made with the students. According to the acquired data, it has been detected that the students experienced difficulties in forming a correct solution set since they acted as if there were no absolute value while finding the solution set of this equation and inequality, and could not fully internalized the concept of absolute value.

Key Words: Artificial intelligence, distance learning, distributed artificially intelligent, web based instruction, agent technology

Introduction

Nowadays, mathematics becomes a nightmare for many students and comes first among the lessons that are considered difficult to learn. Educators have a great responsibility in this respect. The most important one of these responsibilities is to identify the learning difficulties which are experienced about the lesson to be given and to take necessary precautions in view of these identified difficulties. Although covering a very large scope, learning difficulty in mathematics means a number of inadequacies particular to this field (Durmuş, 2007).

Awareness of the difficulties experienced by students in any subject is an important first step for the studies conducted on learning. Synthesizing and correlating such information

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with the subsequent studies will be regarded as a significant basis in arranging the future curricula and in forming the teaching method (Rasmussen, 1998).

The objective of mathematics education is surely to make students actualize learning in the highest level. However, the fact that the vast majority of the students experience difficulties while a few of them actualize learning is considered a reality of life (Tall & Razali, 1993). To identify and eliminate these experienced difficulties; to assist and guide the students during learning process is not only a requirement of modern education but also among the responsibilities of teachers (Ersoy & Ardahan, 2003). Therefore, teachers must be aware of the difficulties that are experienced by students in learning mathematics in order to perform learning activities effectively in their lessons and develop and design the learning environments (Yetkin, 2003).

Tall (1993) stated the reasons for the learning difficulties of students generally as follows: (i) learning the basic concepts inadequately, (ii) inadequacy in formulizing verbal problems mathematically, (iii) insufficiency in algebraic, geometric and trigonometric skills.

On scrutinizing the related literature, it has been found out that educators carried out researches on varied learning difficulties students experience in mathematics beginning from the preschool period up to university level and researches that would help eliminate these difficulties (Coşkun, 2008; Durmuş, 2007; Dikici & İşleyen, 2004; Erbaş, Çetinkaya & Ersoy 2009; Harel, 1989; (Keşan, Kaya and Güvercin, 2010; Özmantar, Bingölbali & Akkoç, 2008; Rasmussen, 1998; Tatar, 2006; Tall & Razali, 1993; Tall, 1993; Ural, 2006).

Tatar and Dikici (2006) determined the learning difficulties of students in binary operation and its properties. Data of this study that they performed on 74 undergraduates was composed of a knowledge test that contained 16 open-ended questions and semi-structured interviews. In view of the study results, it was revealed that the students could not make use of the distributive property in binary operations and experienced difficulty in conceptual level. Moreover, it was observed that the students experienced less difficulty in operations with the table compared to those given with their rules. In the study which was conducted to identify the learning difficulties about the concept of modular arithmetic, Coşkun (2008) observed that students: (i) lack knowledge about division algorithm that forms the basis of modular arithmetic, (ii) experience difficulties in writing the symbolic representation of the division algorithm with modular arithmetic notation, (iii) experience serious difficulties in writing equivalence classes, (iv) confuse any equivalence class with the concept of mod. Çiltaş and Işık (2010) found out the learning difficulties experienced in sequences unit by third-year students who study elementary school mathematics teaching. Data of the study was obtained via 5 open-ended questions and semi-structured interviews made with the students. As a result of the study, it was specified that the students experienced difficulty in finding the limit of a sequence, determining the monotony and boundedness, and finding the limit of a sequence. Furthermore, it was discovered that the students confuse the concept of limit in sequences with that of functions and also confuse the concept of sequence with series.

Baker (1996) conducted his research with the purpose of revealing the difficulties experienced by high school and university students while learning mathematical induction proof technique. As a result of the study, it was brought to light that the students who participated in the research had experienced significant difficulties both conceptually and operationally about proof techniques. It was surmised that students' lack of mathematical knowledge played an important role in these difficulties. It was deduced that many students focused on the procedural aspect of mathematical induction rather than its conceptual aspect. Zachariades, Christou, and Papageorgiou (2002) performed the study, which was aimed to examine the difficulties of students in learning the concept of function, on 38 first-year students who study in the department of mathematics. A test was used in the research, which was composed of 20 open-ended questions that examine whether or not the representations given in each question belong to a function. In the first part of this test are 9 correspondences given in symbolic form and 11 correspondence graphs in the second part. In both parts that are composed of the questions in symbolic and graphical form, it was deduced that there was a statistically significant difference among the difficulties experienced by the students. It was observed that the students recognized the functions more easily in the expressions given symbolically than those given with their graphical representations.

One of the mathematical subjects in which students have the most difficulty is the concept of absolute value. This is clearly observed in the difficulty index study of Tatar, Okur and Tuna (2008) which was conducted with the purpose of detecting learning difficulties in secondary education mathematics. As a result of his study in which he examined the difficulties experienced by 1st grade Turkish-French high school students in the concept of absolute value, Baştürk (2009) observed that the mistakes made by the students in the concept of absolute value are numerous and varied; and the most common mistakes made by them are the ones resulting from the fact that they solved the questions as if there were no absolute value. In their study entitled detection of the difficulties experienced by the elementary school students in absolute value, Yenilmez and Avcu (2009) performed a test composed of open-ended questions on 8th grade students. As a result of the study, it was detected that the students experienced difficulty in the absolute value of expressions with letters and equation questions that contained absolute value. In the study which was performed on the first-year students who study elementary school mathematics teaching, Ciltaş, Işık and Kar (2010) prepared a knowledge test which was composed of open-ended points that examined procedural and conceptual knowledge about the absolute value, and they applied this test on 82 students. According to the data obtained in the research, it was observed that a great majority of the students who participated in the application could not make the geometrical interpretation of the absolute value, and their memorized knowledge from the high school came forward in the procedural test.

Today, it is evident that there are various factors that obstruct students' learning and these factors are compiled in three basic sections as sociological, psychological and cognitive. The aim of this study is to detect the learning difficulties of the secondary school students in equation and inequality that contain terms with absolute values in terms of cognitive factors rather than sociological and psychological factors.

Method

Sample

The sample of the study is composed of a total of 170 volunteer 9th grade students from four different high schools located in Erzurum,Turkey.

Data Collection Tool

“Absolute Value Knowledge Test (AVT)”, which is composed of 10 open-ended questions and which was prepared by benefiting from Horak’s (1994) study, has been used in the study (see Appendix 1). Semi-structured interviews have been made with 10 students in order to detect the learning difficulties in detail. The purpose of this application has been explained to each student in the beginning of the interviews and students’ knowledge on knowledge test has been acquired in a detailed way via expressions like “explain”, “how?” and “why?”. The duration of these interviews has been 10-15 minutes for each student.

Data Analysis

Frequencies and percentage values of the data have been formed by coding the answers of the students given to the AVT as correct, wrong and unanswered. Performed interviews have been transcribed and expressions of the students have been transferred verbatim using descriptive analysis method.

Findings and Discussion

Frequency and percentage distribution of the answers given by the students to the AVT has been listed in Table-1.

Table 1. Frequencies and percentages of the answers given to AVT.

	Question									
	Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**
Correct f(%)	134(79)	81(48)	38(22)	50(29)	10(6)	19(11)	85(50)	30(18)	11(7)	32(19)
Wrong f(%)	36(21)	72(42)	125(74)	115(68)	116(68)	66(39)	78(46)	130(76)	146(85)	100(58)
Unanswered	0(0)	17(10)	7(4)	5(3)	44(26)	85(50)	7(4)	10(6)	13(8)	38(23)

The answers given by the students to the questions in the test and several of the interviews made with the students have been given below.

It has been detected that approximately 14,5% of the students correctly answered the 3rd and the 9th questions in the AVT test; 6% of them did not answer these questions and 79,5% of them gave wrong answers. It has been observed that the ratio of the wrong answers given to these questions which possessed the same feature is considerably high. When these wrong answers have been examined, it has been found out that nearly all students (approximately 90%) answered these questions as in Figure 1. Moreover, the interview which was made with one of the students has been given below.

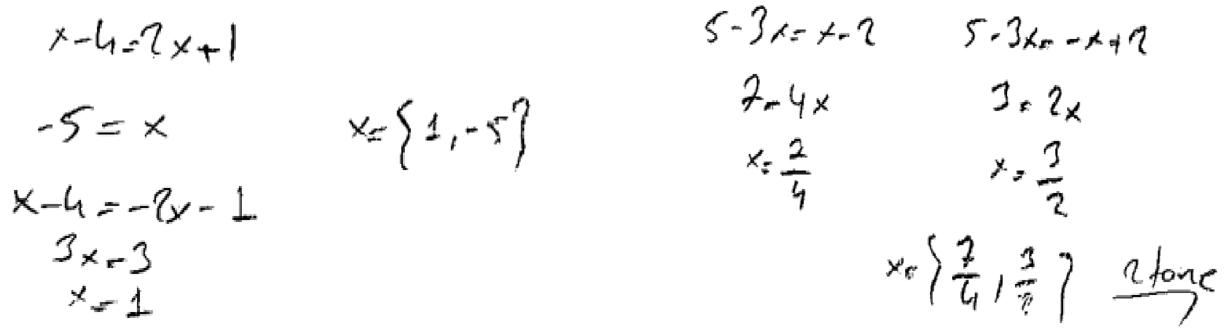

Figure 1. The answers given by a student to the 3rd and the 9th questions.

The interview made with the student who gave these wrong answers has been given below.

Interviewer: Can you tell me how you solved the question "What are the x values that prove the equation $|x-4|=2x+1$?"?

Student: This expression leaves the absolute value as both negative and positive. I found it as positive at first, and then, as negative. When I solved the question, I found x as -5 and 1.

Interviewer: So, have you checked the correctness of the values that you found for this question?

Student: I did not check since I was sure of them.

Interviewer: Can you check them now?

Student: Let's write 1 in place of x; it verifies. Let's write -5 in place of x; it does not verify.

Interviewer: Have you understood why the result that you found does not verify?

Student: No.

Interviewer: What kind of a method that you followed in the other question?

Student: I solved it in a similar way since it was of same logic.

When the 4th and 6th questions in the AVT have been examined, it has been observed that the students experienced a significant difficulty in $|f(x)| = |g(x)|$ type questions. It was detected that they made numerous mistakes while solving this type of questions. Approximately 39,5% of the students correctly answered these two questions. When the answers of the students who gave wrong answers have been examined, interesting answers have been observed. The interview made with the student who answered these two questions as shown in Figure 2 has been given below.

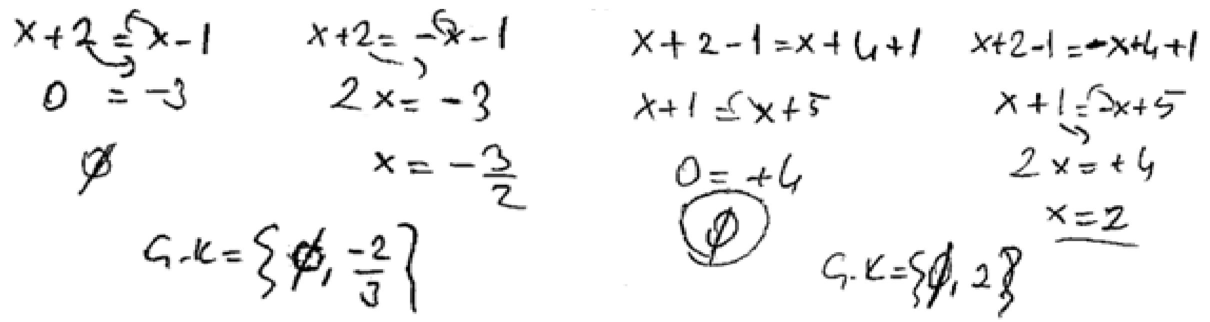

Figure 2. The answers given by a student to the 4th and the 6th questions

Interviewer: Can you tell me how you solved the 4th question in the test?

Student: First, I equated $x+2$ to $x-1$. Then, I equated it to the one with negative value, and I solved it.

Interviewer: Is the negative one of the second expression in the form of $-x-1$?

Student: Yes.

Interviewer: You did the same in the 6th question. So, are you saying that the sign of the variable (that is to say, x) changes, but the sign of the constant does not change?

Student: Yes, we learned so.

Interviewer: So, how have you interpreted the results that you found?

Student: Since -3 cannot be equal to zero in the 4th question, the solution is an empty set.

On the other hand, I found it as -3/2. I wrote it incorrectly in the set there. You asked these as solution set later; I wrote them as solution sets.

Interviewer: You both said that the solution set is an empty set and yet you wrote -3/2. What will you say about this?

Student: I don’t know. I found it so and I wrote it.

Interviewer: If we accepted the empty set result as correct, how would the solution set be written?

Student: I would write only empty set in the set.

It has been observed in the research that a vast majority of the students gave wrong answers or did not give an answer to the 10th question in the AVT as a result of various mistakes and misconceptions. Approximately 30% of the students, who gave wrong answers, did wrong by taking 2 in the denominator to the solution set. They realized their mistakes in the interviews and they interpreted this condition as exam psychology.

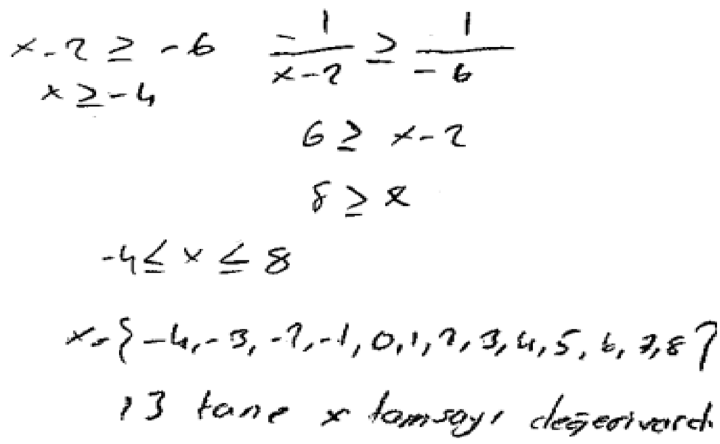

Figure 3. The answer given by a student to the 10th question

The answer of one of the students who wrongly answered this question is shown in Figure 4. Moreover, the interview made with this student is given below.

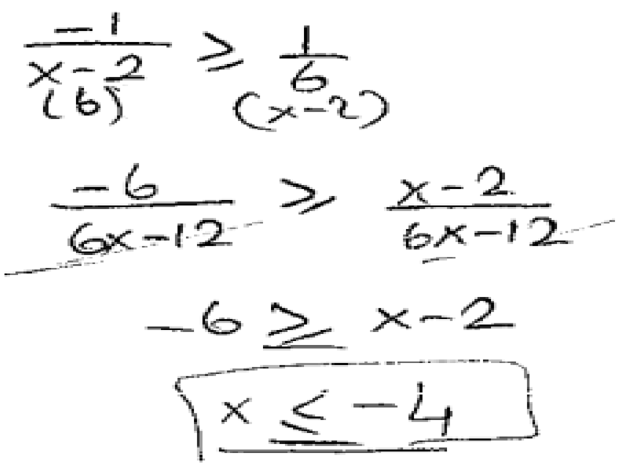

Figure 4. The answer given by a student to the 10th question

Interviewer: Can you tell me how you solved the question "How many x whole number values are there those prove the inequality $\left|\frac{-1}{x-2}\right| \geq \frac{1}{6}$?"?

Student: Firstly, I took it out of absolute value; I equated denominators and reached solution.

Interviewer: So, how many whole number values are there according to this solution?

Student: Whole number values are infinite here since $x \leq -4$.

Conclusions

According the data obtained in the research, it has been observed that a vast majority of the students who participated in the application implied wrong solution methods with the purpose of implementing practical solution methods in order to reach the solution in short time. The fact that the students were in the period of preparing for the university entrance exams can be considered as the reason for this condition. This result is consistent with the study of Çiltaş, et al (2010).

In addition, it has been also observed that they experienced difficulties in solving the inequality that contain terms with absolute value; in properties about the inequality; and in applying the four basic mathematical operations. Furthermore, it has been found out that they wrongfully showed the solution set or they did not show the solution set; did not check whether or not found solution is in the solution set since they did not perform interval analysis; and they experienced difficulties in interpreting the interval that was found correctly in inequality questions. These findings support the results of the studies conducted by Baştürk (2009) and Yenilmez and Avcu (2009).

It was detected both by knowledge test and by performed interviews that the students experienced difficulties especially in the equations in the form of $|f(x)| = |g(x)|$. Data of this research shows consistency with the studies conducted on absolute value and it has been observed that similar difficulties were also experienced in these studies (Çiltaş et al, 2010; Şandır, Ubuz & Argün, 2002).

According to the results obtained in this study, more emphasis should be placed on the concept of absolute value and students must be made comprehend its geometrical interpretation no matter on what level you want to teach the concept of absolute value. This concept should be told using a different method. An education method, in which the student is active and questioning, must be implemented in teaching instead of an education which involves memorization and the students are always passive listeners. When the answers given to the questions and the interviews are scrutinized, it has been observed that the students were made memorize the definition of the concept of absolute value; they could not correctly interpret the concept; and they did not know what it means geometrically. It has been observed that the students had experienced many difficulties in previous subjects and that these difficulties obstructed learning new subjects. For that reason, studies must be conducted, which are oriented to determine and eliminate the learning difficulties in the subjects that require prerequisite. Furthermore, the concept of absolute value must be internalized conceptually and the definition of its geometrical interpretation must be perceived by the students. Moreover, it is considered that the geometrical representation of the absolute value will be very effective in making students comprehend this subject; and programs like Cabri, GeoGebra and Excel can be very useful for that purpose.

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Appendix 1. The Absolute Value Knowledge Test

1. What is the multiplying of the x values that prove the equation $|x-2|=3$?

2. What is the sum of the x values that prove the equation $|3x+2|=|x+2|$?

3. What are the x values that prove the equation $|x-4|=2x+1$?

4. What is the solution set of the equation $|x+2|=|x-1|$?

5. What is the sum of the x values that prove the equation $\left|\frac{x}{2}+2\right|=|3x+5|+3$?

6. What is the solution set of the equation $|x+2|-1=|x+4|+1$?

7. What is the sum of the x integer values that prove the inequality $|3x-2|<5$?

8. What is the sum of the x integer values that prove the inequality $|2x+3|>3$?

9. How many different values of x are there those prove the equation $|5-3x|=x-2$?

10. How many x whole number values are there those prove the inequality $\left|\frac{-1}{x-2}\right|\geq\frac{1}{6}$